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ORDINARY MEETING.

24 January, 1939.

WILLIAM JAMES EAMES BINNIE, M.A., President, in the Chair.

The Council reported that they had recently transferred to the class of

Members.

WILLIAM HERBERT EVANS, B.Sc. (*Witwatersrand*).

CHRISTOPHER HINTON, M.A. (*Cantab.*).

CHARLES HOGG, B.Sc. (*Glas.*).

WILLIAM HUBERT KIRBY, M.C.

FRANK STANLEY MACONACHIE.

PERCY PARR, B.Sc. Tech. (*Manchester*).

ARTHUR CECIL SHARPE, B.Sc. (*Glas.*).

EDWIN CHARLES STEER, B.Sc. (Eng.) (*Lond.*).

REGINALD WILLIAM SCOTT THOMPSON, B.Sc. (Eng.) (*Lond.*).

FRANK WILLCOCK, B.Sc. Tech. (*Manchester*).

And had admitted as

Students.

GEORGE BOTHWELL ALISON, B.Sc. (Eng.) (*Lond.*).

ALFRED DONALD ALSOP.

EDMUND STONES ARMITAGE.

PHILIP EWART BUXTON BAMFORTH.

NEIL ALEXANDER BANNATYNE.

SISIR KUMAR BASU, B.Sc. (*Calcutta*).

HENRY GORDON BEATTIE.

BRIAN MARCUS UNDERWOOD BENNELL.

DAVID BRUCE BRIDGER.

THOMAS KINGSLEY BROUN.

JOHN SHELDON BRYAN.

NORMAN PAUL WHEELER CALCOTT.

ALEXANDER DAVID HOPE CAMPBELL.

FRED NICOL TITT CAMPION.

GEORGE LESLIE COLLYNS.

WILLIAM HENRY COOPER.

DOUGLAS JOHN CRAIG.

JOHN MARTIN JOSEPH CROWE, B.E. (*National*).

GEORGE ALEXANDER DICKIE.

CHARLES COLIN DOCWRA.

CECIL DUTTON, B.Sc. (*Manchester*).

NORMAN BALL DYKE.

JAMES FAIRBAIRN EDGAR.

JACK TREVOR EDWARDS.

DOUGLAS ELLIOTT.

RAYMOND FRANK EMERTON.

DENIS MAIN ROSS ESSON.

ARTHUR HAYTON EVANS.

JOHN COCHRANE FERGUSON.

JOHN BROCKBANK FODEN.

ABDUL GHAFFAR.

GEORGE STANLEY GILL.

JOHN CLEMENT GILLBEE.

JACK GRUNWELL, B.Sc. (*Birmingham*).

PETER JAMES GWATKIN.

PETER SPENCER HALLAS.

CYRIL LEWIS TRAVERS HANNINGTON.

TEODOR-KLEMENS HARTGLAS.

JOHN ROBERT McGREGOR HARVEY, B.Sc. (Eng.) (*Lond.*).

JOSEPH PETER HEWLETT.

DONALD WILLIAM THOMAS HOLLISTER.

DAVID JOHN ELLIOTT HONE, B.Sc. (Eng.) (*Lond.*).
FRANK HOPWOOD, B.Sc. (Eng.) (*Lond.*).
JOHN HORROCKS.
DAVID ASHTON HOWELLS.
WILLIAM WALTON HUFFINGTON.
PETER HASTINGS HYSLOP.
WILLIAM RALPH COLEMAN IVENS.
GREGORY EDWARD ANTON JAYA-WARDANA.
RICHARD HAMBLET BLAKE JOHNSON.
DOUGLAS HENRY KING.
LOUIS PATRICK KIRWAN.
ERIC KNIGHT.
RICHARD ARTHUR EDGAR LAMMING.
MICHAEL WEMYSS FLOCKHART LEBURN.
WILLIAM GORDON CARLYLE LOVATT.
JOHN DANFORD DE SALIS McELWAIN.
MICHAEL STEWART McFIE.
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WILLIAM GEORGE MARTIN.
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GEORGE HOLMES MELVILLE-JACKSON.
ERIC JOSEPH MILES.
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RONALD DAVID MONTGOMERY, B.Sc. (*Edin.*).
JOHN HENRY MORRISON.
MICHAEL JAMES MORTON.
RICHARD NEAVERSON, B.Eng. (*Liverpool*).
HAROLD PATRICK ADAIR NELSON.
JOHN WILLOGHBY NEVE.
PETER FORBES NICHOLSON.
JOHN WILLIAM THOMSON NORQUAY.
THOMAS SEELEY OLDHAM.
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THILLAIAMPALAM VISUVALINGAM PERU-MAINAR.
GORDON BEGG RATTRAY, B.Sc. (*Cape Town*).
JOHN ROBB, B.Sc. (*Edin.*).
JOHN CECIL ROBINSON.
PETER JOHN MACKENZIE ROBINSON, B.Sc. (Eng.) (*Lond.*).
FREDERICK STUART RODGERS.
ALFRED JOHN ADAMS ROSEVEARE.
ALEXANDER HENDERSON RUTHERFORD.
DEVI KRISHNA SAHGAL, B.Sc. (*Lucknow*).
GERALD MICHAEL SEFTON.
KSHITINDRA NATH SEN, B.Sc. (*Calcutta*).
FRED SHARRATT.
WILLIAM JEFFERSON SMITH.
WILLIAM SPENCER.
ROLAND STEVEN.
JOHN STORRY.
SAJJA VENKATA SUBBARAO, B.A. (*Andhra*).
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VIVIAN PRYCE THOMAS.
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HAROLD DOUGLAS THORNLEY.
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FRANK TRISTRAM.
JOHN PEACE TWATT, B.Sc. (*Edin.*).
PETER TWOMEY.
ROBERT LESLIE VALENTINE.
MAURICE PERCY WALKER.
KENNETH EDGAR WATKINS.
HERBERT RAMSEY WATSON.
PETER GORDON WAUGH.
STANLEY HUGH WESTON.
CLIFFORD GORDON WHITING.
RONALD MacKINNON WILKIE.
GEORGE HOWARD WILKINS.
FREDERICK WINPENNY.
CHARLES WOOD.
JAMES WOOD.

The following Paper was submitted for discussion, and, on the motion of Sir Clement Hindley, Vice-President, the thanks of The Institution were accorded to the Authors.

Paper No. 5188.

"The Gorge Dam."†

By WILLIAM JAMES EAMES BINNIE, M.A., and
HAROLD JOHN FREDERICK GOURLEY, M. Eng., MM. Inst. C.E.

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INTRODUCTION.

THE Jubilee reservoir, which was formally opened by His Excellency Sir Andrew Caldecott, K.C.M.G., C.B.E., the Governor of the Colony of Hong Kong, on January 30th, 1937, is formed by means of two dams, one known as the Gorge dam, being the main structure placed across a narrow gorge through which the Shing Mun river flowed, and the other known as the Pineapple Pass dam, across a depression in the ridge which forms the boundary of the catchment area. The design and construction of the Gorge dam and ancillary works form the subject of this Paper.

HISTORICAL.

Hong Kong, the capital of which is Victoria, is an island separated from the mainland of China by an arm of the sea (Fig. 1, Plate 1) about

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† Correspondence on this Paper can be accepted until the 15th June, 1939.—SEC.
INST. C.E.

1 mile in width at its narrowest point.   The Colony embraces three areas;
two of them, the Island and Kowloon on the coast of China, are British
possessions, and the third, an additional area on the mainland, was leased
from the Chinese Government in 1898 for 99 years, and is called the
" leased territory."

With the exception of the comparatively level areas on which parts of
the towns of Victoria and Kowloon are situated, the country is mountainous,
the highest peak on the Island being about 2,000 feet and on the mainland
3,000 feet.

The streams flow to the sea through narrow steeply-inclined valleys,
so that the problem of providing adequate water storage is a difficult one
as the capacity of the reservoirs is small compared with the height of the
dams required to form them.

The consequence has been that though eight reservoirs in the Island,
having a gross capacity of 2,360 million gallons, and four in the mainland,
having a gross capacity of 687 million gallons, were constructed from time
to time, there have only been 7 years since 1921, prior to the completion
of the Gorge dam, when an unrestricted supply of water could be given
throughout the year, and in some years the shortage was acute.

In addition to the supply which was derived from storage, a weir and
intake had been constructed on the Shing Mun river on the mainland,
whereby water was taken from that stream and conveyed into the Kowloon
reservoir by means of an open conduit and two tunnels.

Hong Kong is so situated that, although the rainfall is high—about
100 inches per annum—rain only falls during the monsoon period, lasting
about six months, the remainder of the year being almost rainless, with the
consequence that towards the end of the dry season, this source could
not be depended upon to yield more than about 2 million gallons per
diem.

Faced with the necessity of obtaining further sources of supply, Mr.
R. M. Henderson, C.B.E., M. Inst. C.E., Director of Public Works, who
was then Chief of the Water department, made an exhaustive study of all
possibilities, and finally came to the conclusion that the best solution
would be to construct a reservoir or reservoirs impounding the waters of
the Shing Mun river.

The catchment area to the site of the proposed dam is limited in extent,
amounting to about 3,000 acres, but catchwaters, discharging by gravity
into the reservoir, would secure the inclusion of the runoff from an additional
area of greater extent.

The reservoir would be at such an elevation as to enable the water to
be discharged by gravity into the conduit which already conveyed the
water from the intake to the Kowloon reservoir, and it was estimated that
the construction of the reservoir would ensure a supply from the Shing
Mun river of 9¼ million gallons per diem derived from the catchment area
draining directly into the reservoir, which would be increased to 14 million

gallons per diem when the first catchwater discharging into the reservoir at Pineapple Pass dam was constructed.

The additional supply is required not only for the mainland but also for the Island, being conveyed thereto by means of two mains, one 12 inches and the other 18 inches in diameter, which have been laid on the bed of the channel separating Kowloon from Victoria.

Before giving effect to Mr. Henderson's recommendations, it was decided that the services of the Authors' firm should be requisitioned to consider and report on the various alternatives which had been suggested, and in accordance with this decision a visit was paid at the end of 1930, and the conclusion was arrived at that the total resources capable of economical development on the Island would be exhausted when certain works which were then in progress had been completed, and that further sources would have to be sought on the mainland, where, after reviewing various alternatives, it appeared that the best solution was to construct a reservoir on the Shing Mun river.

In order to provide sufficient storage, having in view the additional catchment area which would ultimately contribute to the reservoir, it would be necessary to construct a dam of very considerable height, and whether this was economically feasible would depend upon suitable foundations being secured at a reasonable depth below the surface.

It was therefore recommended that exploratory work should be carried out by the Public Works department, on the conclusion of which another visit would be paid with the object of determining whether or not the construction of the dam was an economical proposition, and to locate its position.

EXPLORATORY WORK.

The rock, where exposed at the gorge through which the stream flowed and in the bed of the river just above, was a fine-grained, moderately basic, biotite-hornblende granite. The sound rock, weighing 165 lb. per cubic foot, crushed at a load of about 2,000 tons per square foot.

The river fell rapidly through the gorge, and the height of the dam would be considerably reduced if it were located just above, its length being not materially increased, so investigations commenced by excavating trenches which were to be carried down to rock level on each side of the river on a centre line which had been provisionally selected during the first visit.

Rock was exposed in the bed of the river at this point, but the trenches indicated that it was covered by decomposed granite material to a depth which increased in proportion to the height above stream-bed level, and when the second visit was paid in October, 1931, rock had not been reached at the upper end of the trenches, and work had been suspended as it had been deemed impracticable to proceed without heavy timbering.

Boreholes had, however, been sunk in line with the trench above top

water-level, which indicated that sound rock would be reached at a depth which was not excessive.

It was therefore decided that it would be economically possible to construct a high masonry dam at this site, and the next consideration was the maximum level to which water could be safely impounded, as it was advisable to store as much water as possible at this site.

It was originally intended to construct a dam which would raise the water-level to 600 feet above O.D., when the capacity of the reservoir would have been approximately 2,000 million gallons, but by increasing the level to 625 O.D. the capacity would be increased to 3,000 million gallons, having about the same contents as the total provided by the twelve reservoirs previously constructed.

The ridge which separates the Shing Mun river from the stream flowing into Gin Drinkers bay is narrow, falling steeply on both sides, and the top of this ridge was traversed by a " col " called Pineapple Pass (Fig. 1, Plate 1), where a subsidiary dam about 90 feet high and 470 feet long was necessary, and there were other depressions which would involve expensive works if the top water-level were fixed at too high a level.

A complete contour survey of the reservoir area had been made under the direction of Mr. Henderson, and after additional levels had been taken along the ridge, it was decided during the second visit that 625 O.D. was the maximum economic top water-level, but the selection of the exact location of the dam and its design were held to be premature until further exploratory work had been undertaken.

It had been decided to carry out the construction by administration, the Authors' firm being appointed to design and supervise the work, and Mr. G. B. G. Hull, M. Inst. C.E., being appointed as Resident Engineer to have charge of the construction.

Mr. Hull commenced his duties at the end of 1932, and during the year 1933 numerous boreholes were sunk in order to determine the sub-surface contours of the top of the rock.

During the course of drilling it was discovered that there was a zone of decomposition lying at a depth of about 50 feet below the surface at the bottom of the valley, the granite being converted into a permeable sandy layer, and it was therefore obvious that foundations would have to be carried to a great depth, if the dam were located above the gorge, in order to prevent the escape of water which might reach this layer through joints in the rock.

It was therefore decided to abandon all thought of locating the dam above the waterfalls and to build it across the gorge at the point shown on *Fig. 2*, where rock was exposed on the left bank to a level which left no doubt that it would be found at or near the surface on that side of the river, and on the right bank to about 80 feet above the river bed, which was encumbered with boulders.

This necessitated an increased height of about 40 feet in order to obtain

Fig. 2.
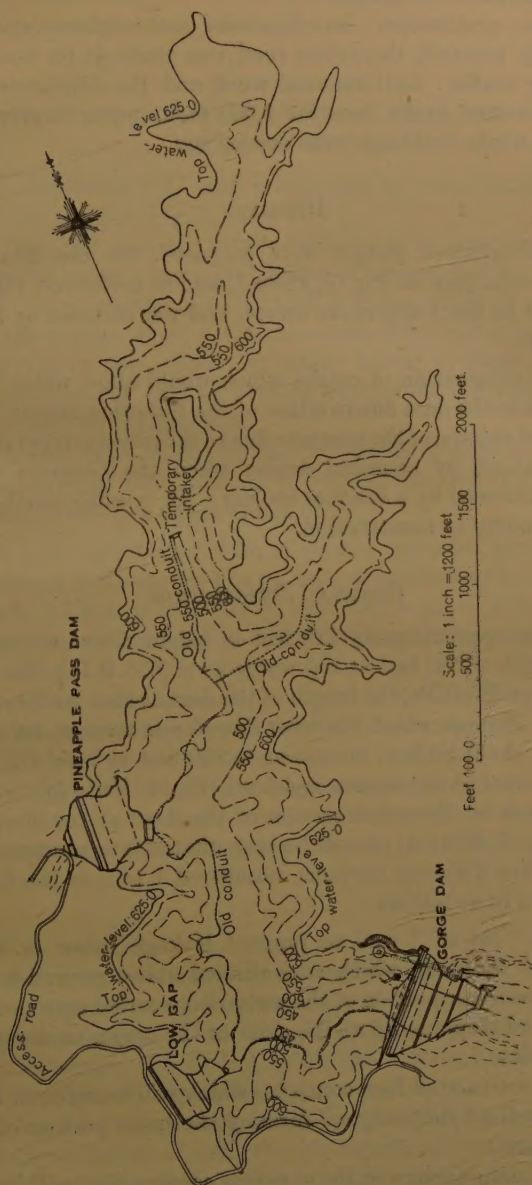
PLAN OF JUBILEE RESERVOIR.

a top water-level of 625 O.D., but the dam would be considerably shorter than if located above the gorge.

While the preliminary investigations and exploratory work were being actively pursued, the access road was made fit for heavy and continuous lorry traffic ; anti-malarial work and the construction of staff- and office-quarters, coolie lines for 2,500 men, water supply, sanitation, hospital and works buildings were carried out.

DESIGN.

Various alternative designs were prepared, the one finally selected, which is shown in plan on Fig. 7, Plate 1, and in section on Fig. 3, Plate 1, is novel so far as the Authors are aware, and was dictated by the following considerations :

(a) The elimination of cracks which would allow water to penetrate into the dam due to either of the following causes : (1) changes of volume of the concrete due to the rise in temperature brought about by chemical action when the concrete was setting, followed by a gradual cooling ; (2) possible earth-tremors.

(b) Economy in construction.

GENERAL DESCRIPTION.

The dam is approximately 275 feet in height above stream bed-level, and has a length of 690 feet at top water-level (625 O.D.), the roadway on the top being at 635 O.D., the length of the dam at that level being 700 feet.

The gorge through which the river flowed was narrow, its width at the bottom being about 50 feet, increasing to 120 feet at level 453 O.D. ; that is at about 93 feet above stream bed-level (Fig. 4, Plate 1).

On reference to the cross section of the dam at the deepest part of the gorge (Fig. 3, Plate 1), it will be seen that it is a composite structure consisting of five different portions, numbered 1 to 5 in the figure, which will be referred to as follows :

(1) The " cut-off " wall to prevent leakage below or at the sides of the dam, which is continued above ground-level to form the lower portion of the water-face carried up to level 453 O.D. and stepping up at the sides of the gorge, as shown on Fig. 3, Plate 1.

(2) The articulated face concrete, which will be referred to hereafter as the " diaphragm," forming the main portion of the water-face.

(3) The main portion of the concrete construction will be referred to as the " thrust-block," its function being to transmit water pressure to the rock-fill and to act as a retaining wall when the reservoir is drawn down.

(4) The "rock-fill," which takes a portion of the water-pressure.

(5) The "sand-wedge," which is interposed between the thrust-block and the rock-fill.

The Cut-off Wall and Water-Face of the Lower Portion of the Dam.

This wall is numbered " 1 " and is indicated by cross hatching inclined downwards to the right on Fig. 3, Plate 1.

Provided that the concrete at the water-face is impermeable, which can be ensured by using a large proportion of cement and by careful proportioning of the aggregate, placing and working, there is no necessity to construct the remainder of a concrete dam of so rich a mixture, with consequent saving in cost, especially where labour is cheap, as at Hong Kong, and therefore the cost of cement represents a considerable proportion of the total cost of concrete.

It is well known that the larger the proportion of cement, the greater will be the amount of heat evolved in setting, the following being the results of actual experience at the Gorge dam :—

A concrete mix containing 600 lb. of cement per cubic yard of concrete showed an average temperature-rise on setting amounting to 53° F. above air temperature, whereas if only 300 lb. were used—which is the proportion specified for the thrust-block to be described later—the temperature-rise amounted to only 24° F.

Where, therefore, a rich concrete is placed in contact with a poor concrete so that the mixtures cohere, considerable stresses will be set up in cooling and the rich concrete-mixture will tend to crack.

It was therefore decided to introduce greased paper so as to form a vertical joint separating the face concrete from that of the thrust-block, to prevent the two concretes from cohering and to allow of differential settlement.

Excavation.

The rock had been subjected to surface decomposition except at the bottom of the gorge, the overlying material, which consisted of sandy clay, attaining considerable depths on the right bank of the river, sound rock lying sometimes as much as 40 feet below the surface, its position being shown by a dotted line on Fig. 4, Plate 1.

At the surface, the rock was generally found to be pervious and brown in colour due to oxidation of iron, though capable of bearing a considerable load. The quality and strength of the rock rapidly improved as the trench was sunk, until a satisfactory foundation was reached in the grey granite.

Granite, an igneous rock, contracts as it cools, so that it is seldom free

from cracks, and is traversed by major joints where the infiltration of water has brought about decomposition, the product filling the joint being generally either a mixture of flakes of mica or less decomposed rock and clay or china clay; but occasionally the joints are not completely filled or may be filled with material nearly as permeable as sand.

As excavation proceeds, the rock improves, but it is not probable that a foundation free from all cracks or joints would ever be reached, it being known that decomposition has affected granite even at depths of over 900 feet below the surface.

An economic limit had therefore to be fixed and instructions were given that excavation was to proceed until rock was reached which gave promise of a sound foundation, when boreholes were to be drilled to a depth of 20 feet, which were to be tested under a water-pressure of 100 lb. per square inch, and should this test show a leakage of more than one-fifteenth of a gallon per minute per foot depth of hole during a test of 7 minutes duration, the trench was to be sunk to a deeper level and the test repeated until this condition was fulfilled.

The tests showed that the rock, generally speaking, was watertight, but at one place at level 550 on the left bank, a bad joint was encountered, and measurements of direction and angle of dip showed that this joint was a continuation of one exposed in the tunnel excavation below. A pit was sunk at this point to a depth of 40 feet below the bottom of the trench and drill-holes to a further depth of 20 feet, which were afterwards grouted. Pipes were brought up from rock level on the downstream side of the cut-off wall to discharge any water which might pass below this wall into the drainage gallery shown on Fig. 3, Plate 1.

The depth to which the cut-off trench was carried is shown by a dotted line on Fig. 4, Plate 1.

Concrete.

There are no contraction- or expansion-joints in the face-concrete, which was poured in blocks facilitating the more rapid dissipation of heat, the blocks being so arranged that there were no continuous joints through which water might find its way.

No apprehension was felt with regard to the effect of changes of temperature once the reservoir was full, as the face-concrete would either be almost entirely below ground or water-level, the lowest level (485 O.D.) to which the reservoir could be drawn down so as to discharge into the supply conduit by gravity being well above that of the top of the face-concrete in the gorge (453 O.D.).

It was considered possible that some cracks might appear during construction which could be dealt with before the reservoir was filled, and therefore a careful watch was kept, but the only crack which could be discovered was a horizontal hair-crack on the right side of the gorge, which afterwards closed up.

It was decided to bring up the concrete in the gorge in 20-foot lifts, and as experience has shown that it is difficult to secure a watertight junction between concrete which has already set and green concrete poured above it, it was arranged that, in addition to the usual " joggled " mortar joint, strips of copper $\frac{1}{16}$ inch thick should be built into the top of each lift. These strips were 2 feet in width and extended across the gorge, the lower half being built in at the top of each lift, the upper half projecting upwards into the concrete above.

The strips came in 25-foot lengths and the junction of the ends of the strips was made by means of an overlap of 7 inches, the ends of the strips being smeared with bitumen and bolted together by means of small brass bolts screwed up until the bitumen was extruded.

The " face-concrete " has a batter of 3·4 vertical to 1 horizontal, being 6·3 feet in thickness where it terminates at level 453 O.D. and 33·7 feet at the bottom of the gorge (360 O.D.).

It will be noticed from Fig. 3, Plate 1, that the batter is interrupted by a series of steps at the top of each 20-foot lift, the face being brought up vertically so as to form a ledge 2 feet in width in order to facilitate the setting of the shuttering, and these steps proved a great convenience.

The aggregate consisted of the following proportions, determined by experiments carried out to discover the mix which gave the greatest impermeability : 16$\frac{1}{4}$ cubic feet of sand, half of which was coarse sea-sand, the remainder being fine sand and dust—the product of the crusher—and 23 cubic feet of rock crushed to pass through jaws set to 2 inches, to which was added 600 lb. of cement to form 1 cubic yard of concrete in place.

The amount of water was adjusted to suit climatic conditions so as to obtain a dry mix giving a slump of $\frac{7}{8}$ inch, and the concrete was spread in 6-inch layers and vibrated by means of " Broomwade " pneumatic punners, the concrete at 28 days crushing at about 260 tons per square foot.

When the dam was completed it was found that the slip-joint separating the concrete of the water-face from the thrust-block had opened up, due no doubt to the contraction of the concrete when cooling down to atmospheric temperature.

The open joint so formed proved to offer the path of least resistance for water percolating below the cut-off wall, this water discharging into the galleries in the thrust-block where it was collected and measured over a V-notch just before passing into the rock-fill through the drainage pipe (Fig. 3, Plate 1).

The amount of leakage water since impounding has varied between about 20 gallons and 430 gallons per hour, and there is no direct relationship with the level of the water in the reservoir, the leakage being dependent on certain slight movements affecting the lower portion of the main structure, which will be referred to later, as these movements caused the width of the joint between the facework and the thrust-block to vary slightly.
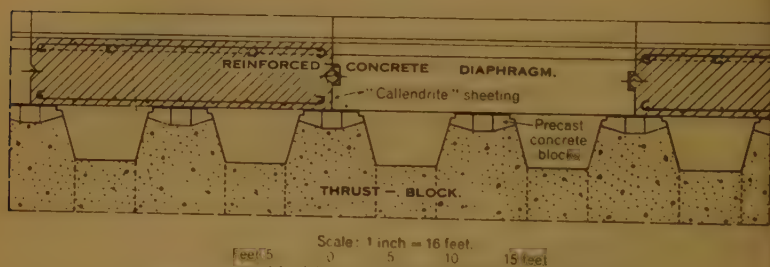
The Diaphragm.

The diaphragm, which forms the main water-face, is shown by hatched lines inclined downwards to the left on Fig. 3, Plate 1, where it is numbered "2." This is an articulated structure divided into vertical panels 25 feet in width, as shown on Fig. 4, Plate 1.

Each panel was brought up in "lifts" of 20 feet in one operation, and is reinforced with horizontal hooked rods wired to verticals, so arranged as to form a "grille" (*Figs.* 5 and 6) the rods being $1\frac{1}{4}$ inch in diameter, the spacing varying with the maximum water-pressure from 9 inches to 2 feet.

The diaphragm is 6 feet thick at the level 453 O.D., reducing to 3 feet at the level 573 O.D. and continuing at that thickness to the top.

Fig. 5.

REINFORCED · CONCRETE   DIAPHRAGM.

"Callendrite" sheeting

Precast concrete block

THRUST — BLOCK.

Scale: 1 inch = 16 feet.

SECTION THROUGH BUTTRESSES AND DIAPHRAGM.

The general face-batter is 3·4 to 1 up to the latter level and 3·72 to 1 above, the buttresses against which it rests having a batter of 3·72 to 1 throughout, the minimum thickness being determined by the width necessary for men to place and vibrate the concrete when working between the shuttering.

As the cement was slow-setting and as the concrete was to be poured to a height of 20 feet in one operation, it became necessary to carry out experiments before the forms were designed, to ascertain the lateral pressure which would be exerted by the green concrete in a semi-liquid condition.

The panels were cast alternately, and the steel skin of the forms was supported by lattice girders of the queen-post type, the foot of the posts being inserted in a groove in the step (*Figs.* 6), which was filled with asphalt (bitumen plus 30 per cent. granite dust) when the form had been removed.

The top of the queen-post extended beyond the form and was tied back to the thrust-block, temporary timber struts being inserted to give the correct width and removed as the concrete was brought up.

The diaphragm rests against buttresses, protruding from the thrust-block (to be presently described), being separated therefrom by a layer of

" Callendrite " bitumen sheeting, $\frac{1}{5}$ inch thick, specially manufactured
to stand the maximum pressure to which it would be subjected without
appreciable compression, thus forming a sliding joint.

The concrete was poured directly against this sheeting, and timber
shuttering was employed to form the void between the buttresses, the ends
of the shutters being housed in grooves in the moulded concrete-blocks
with which the buttresses were faced (*Fig.* 5 and section XX, *Figs.* 6)
in order to acquire exact alignment of the face against which the diaphragm
rests.

Figs. 6.

CROSS SECTION OF DIAPHRAGM.
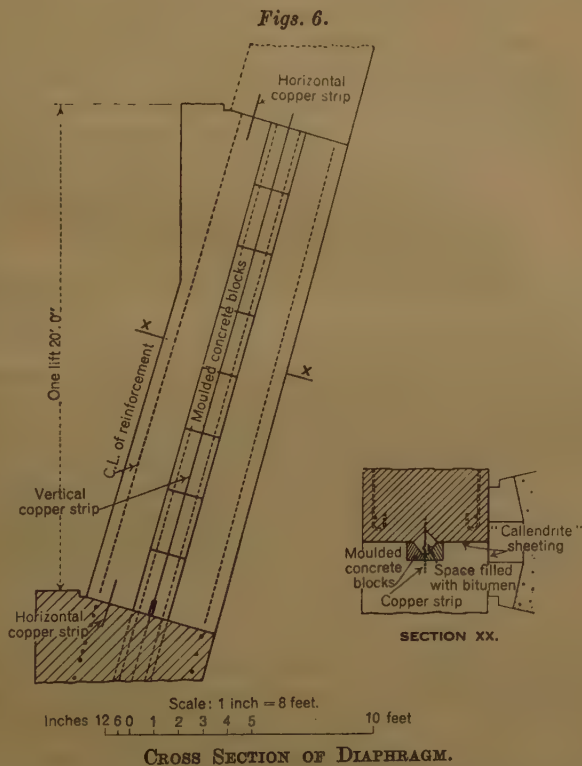
The steps shown on *Figs. 6,* and already described, proved of great
benefit in construction as the steel forms could be readily placed by means
of a cantilever crane carried by a 4-foot 8-inch track on top of the thrust-
block, which was brought up in advance, and room was given for the
workmen to set the forms accurately, a " cage " being provided to prevent
accidents.

" Callendrite " sheeting was placed in vertical strips between each
panel and also where the panels butt against the solid portion of the dam.
The bottom of each panel was also separated from the solid " face-concrete "

on which it rested by a strip of similar sheeting, the percolation of water through this joint being prevented by a 12 S.W.G. copper strip built into the solid face-concrete below and carried up into the diaphragm, the width of the strip varying from 2 feet to 1 foot according to the water-pressure.

A similar copper strip was provided at the top of each 20-foot lift (*Figs. 6*), the " Callendrite " sheeting being omitted between the lifts.

The object of inserting this sheeting at the base of each panel was to permit slight rotation about a horizontal axis without injury to the diaphragm should some slight relative movement of the thrust-block take place.

The vertical joints between the panels (which are shown in *Fig.* and in more detail on section XX, *Figs. 6*) are formed by similar copper strips " kinked " in the middle, varying in width from 27 inches to 19 inches according to the water-pressure, being built into the solid face-concrete below and extending upwards in 20-foot lifts, the joints being made by an overlap of 7 inches, the adjacent faces of the overlap being smeared with bitumen and the strips bolted together with small brass bolts until the bitumen was extruded.

A similar vertical joint is made where the panels butt against the solid face-concrete.

A " void," which is traversed by the copper strip dividing it into two halves, was also formed at all vertical joints and is filled with asphalt (bitumen plus 30 per cent. granite dust).

Fig. 5 is a section along a plane at right angles to the face of the diaphragm, which shows the voids which have been formed each side of the copper strip at the end of the two panels which have been already cast.

The concrete is about to be poured to form the intermediate panel and is to be brought up rapidly to a height of 20 feet, it being necessary to make provision for the remaining portion of the " voids."

As it was thought that there would be great, if not insuperable difficulty in removing the " cores " from such confined spaces, it was decided to use small moulded concrete-blocks so shaped as to form the voids, being incorporated with the concrete as it is brought up (see detail, section XX, *Figs. 6*).

Two blocks were used, one on each side of the copper strip, the column of blocks being brought up to full height before the concrete was poured, the joint being made between the strip and the block with Portland cement mortar with the exception of that between the lowest pair of blocks.

If good bond is to take place between successive lifts of asphalt it is essential that the surface of contact should be clean and dry, and therefore the space between the lowest pair of blocks was $2\frac{1}{2}$ inches wide, through which dust, etc., could be expelled by compressed air, after which this space was also filled with mortar, the top of the column being covered with a timber cap to prevent entry of dust, etc., until pouring of asphalt commenced.

The columns having been built to their full height, the reinforcement was set and the concrete poured into the intermediate panel, after which operation the caps were removed, the surface of the asphalt at the base of the column brought to a liquid state by electrical heaters and the void filled with asphalt.

The concrete mix, which was vibrated as already described, contained 690 lb. of cement added to 15½ cubic feet of sand and 22½ cubic feet of crushed granite to form 1 cubic yard in place, the granite being broken so as to pass the jaws of the crusher set with a clearance of 1½ inch. The average crushing strength at 28 days amounting to 350 tons per square foot.

After the forms were stripped the concrete was kept moist for a minimum period of 3 weeks by means of water sprays.

The diaphragm-concrete and the joints described above have proved to be remarkably watertight, there being no leakage when the reservoir was filled to top water-level.

THE THRUST-BLOCK.

The thrust-block is indicated by blobs and dots, and is numbered " 3 " on Fig. 3, Plate 1.

The function of this portion of the dam is to transmit part of the water-pressure to the rock-fill to be presently described, and it was therefore not considered necessary to carry down the foundations below the level of the surface of the sound rock, the depth to which they were carried being indicated by a dot-and-dash line on Fig. 4, Plate 1.

The width at the top is 9·5 feet measured across the buttresses, for the central portion of the dam (Fig. 7, Plate 1), widened at each end to 18 feet 6 inches in order to accommodate the steps leading to the galleries and to 27 feet 6 inches at the siphons, to be presently described, which are located at the north end of the dam.

The back slopes downwards at an inclination of 1 horizontal to 9 vertical, for the central portion, the face-batter being 3·72 to 1 down to a level which varied with ground-level, the maximum width at the base being 118 feet.

Fig. 5 shows the buttresses which are spaced at 12-foot 6-inch centres and protruding 4 feet 6 inches, the width being 5 feet at the diaphragm, increasing to 7 feet 6 inches at the " root," leaving an inspection pit between each buttress having a cross-sectional area equal to that of the buttress, access to the bottom of the pits being provided by means of branch galleries connecting with main galleries, which are reached by means of steps from both ends of the dam (see Figs. 3 and 4, Plate 1).

The inspection pits are provided with hinged covers giving a clear opening of 39 inches by 33 inches, thus affording the possibility of inspection of any portion of the joints between the buttresses and the diaphragm.

A transverse gallery, which is shown on Fig. 3, Plate 1, conveys any

leakage water to a sump, a pipe being provided to carry the water into the rock-fill, and a ventilating pipe 18 inches in diameter is carried to the top of the dam, this pipe being also used for taking observations of movements of the dam by means of a " plumb-line."

The top of the thrust-block oversails the diaphragm, as shown on *Fig. 8*, a space of 1 inch being left to allow free expansion and contraction of the latter.

The parapet-wall is carried on this oversailing portion which is reinforced for that purpose, and holes are provided for the purpose of feeding bitumen into the vertical diaphragm-joints should it be found necessary to do so.

The concrete-mix contained 300 lb. of cement added to 16·25 cubic feet of sand and 23 cubic feet of stone (2½ inches) to form 1 cubic yard in situ, and " plums " were embedded, the concrete being vibrated as already described, the average crushing strength at 28 days being 190 tons per square foot.

THE ROCK-FILL.

The rock-fill is numbered " 4 " on Fig. 3, Plate 1.

During a visit to Japan the effects of the earthquake of 1923 on dams for impounding water for Tokio and Yokohama were studied and the conclusion reached was that the effect of an earthquake on a solid masonry structure may be serious, and that an embankment formed of non-cohesive material was more suitable for impounding water where violent earth-tremors may occur.

Owing to the great height of the Gorge dam, it was decided to adopt rock-fill for the support of the thrust-block, as this material could be obtained in abundance from quarries immediately below the dam and could be economically handpacked owing to the cheapness of the Chinese labour. Another reason for its adoption was that should water find its way into an earth-embankment the cohesion of the material might be so reduced as to cause slips, especially when it is borne in mind that the bank would be of unprecedented height above the toe as the river-bed falls steeply through and below the gorge.

The rock-fill had a face-batter of 1 vertical in 1·5 horizontal, benchings 10 feet in width being formed at levels 461 and 550 O.D., and terminated in a plateau 40 feet in width at level 380 O.D., supported by a retaining wall. A benching 18 feet in width is also provided at level 622 O.D., the maximum width at the base being 440 feet.

The stones were handpacked in layers 2 feet in thickness with an inclination of 1 in 12 towards the thrust-block, the interstices between the larger stones being filled with smaller ones, and the average weight per unit of volume was two-thirds of that of solid granite.

It was not thought necessary to remove the material consisting of sand

Fig. 8.
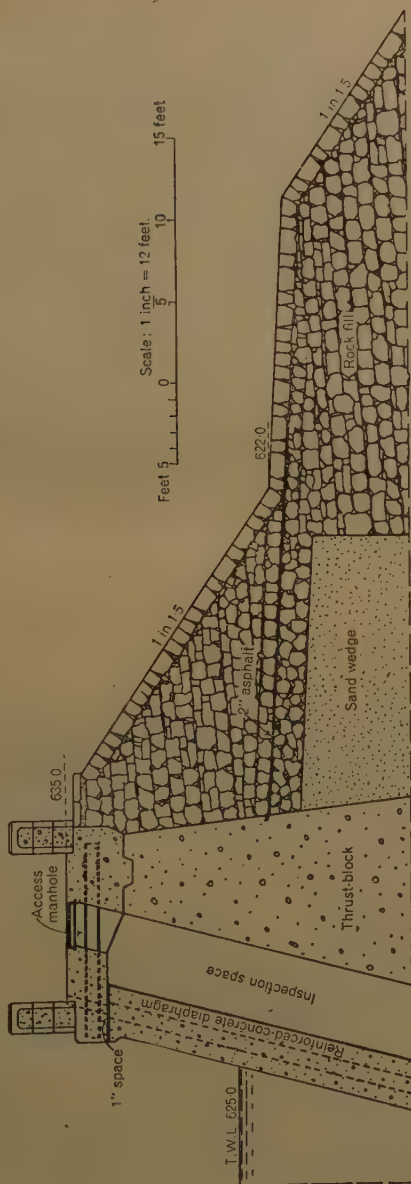
SECTION OF TOP OF DAM.

13

and boulders which overlaid the solid rock in the stream-bed, so that the rock-fill rests on this material, on solid rock, and on the stripped ground at the sides of the valley.

It was desired to ascertain the amount of compression of the foundation-material which would take place, and plates were therefore placed in the rock-fill connected to vertical steel-rods which were lengthened as the fill was brought up, the levels of the plates being ascertained before loading took place.

As the length of the rods was known, it was possible by taking levels of the top of the rods to ascertain to what extent the plates had sunk, showing the compression of the foundation-material and the rock-fill below the plates. This compression proved to be more than had been expected, and has been dealt with by one of the Authors when discussing the Paper on the Subsidence of the Eildon Reservoir Dam [1]. It attained a maximum of about 4 feet at the centre of the gorge.

The handpacked rock-fill did not, however, appear to compact vertically to any great extent, the evidence afforded by the plates pointing to less than 6 inches.

Levels were taken on the benchings which showed that settlement ceased in June, 1937.

The rock-fill is faced with squared granite blocks, 14 inches by 14 inches by 12 inches thick, and owing to the irregular settlement which was taking place it was decided to lay these blocks in zig-zag courses which gave a very pleasing appearance, as shown in *Fig. 9*.

The rock-fill is separated from the thrust-block by the sand-wedge, to be presently described, and the upstream face which is vertical above the bottom of the sand-wedge was built with mortar joints to prevent the sand entering the interstices, drainage tubes, consisting of pipes 4 inches in diameter filled with pea-gravel kept in place by screens at each end, passing through the wall at intervals of 10 feet with the object of affording drainage should rainwater find its way into the sand-wedge.

It will be seen from *Fig. 8* that the sand-wedge is brought up to level 620 O.D. and is covered by the rock-fill above that level.

THE SAND-WEDGE.

The triangular space between the rock-fill and the thrust-block is filled with sand, numbered " 5 " on Fig. 3, Plate 1, the purpose being to avoid point-contact and to ensure that the water-pressure is transmitted to the rock-fill should movement of the thrust-block later take place.

It was specified that the sand should be a clean, coarse, dry sand so as to settle readily should any movement take place of the rock-fill.
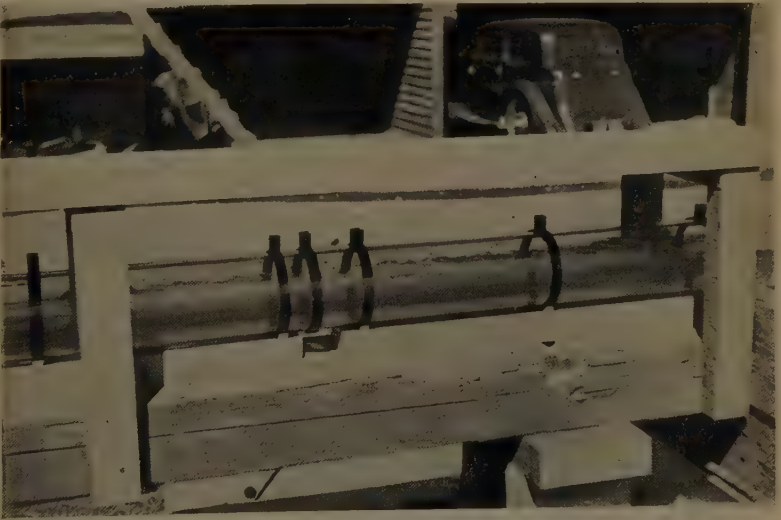
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[1] *Discussion on* " The Subsidence of a Rockfill Dam and the Remedial Measures employed at Eildon Reservoir, Australia." Journal Inst. C.E., vol. 8 (1937–38), p. 192. (March 1938.)
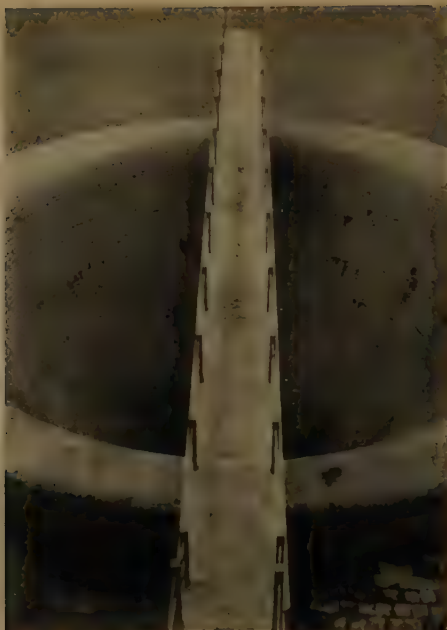
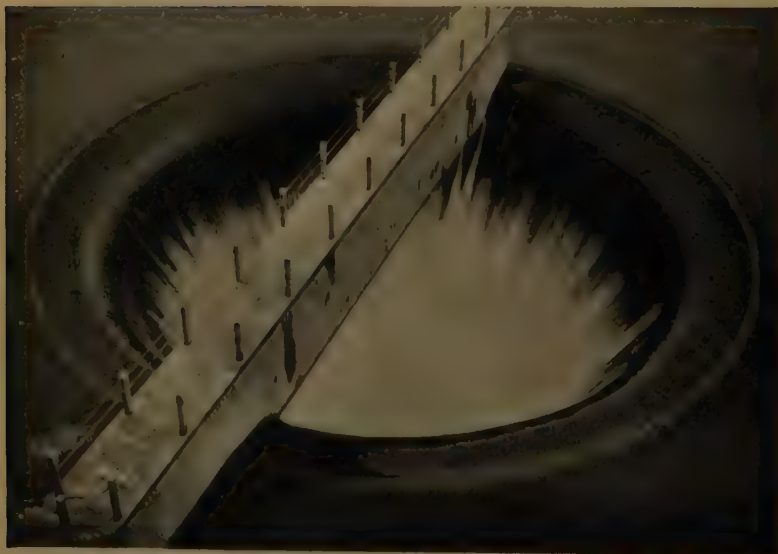Fig. 9.

PITCHING ON DOWNSTREAM FACE OF THE DAM.

Fig. 17.

CELLULOID MODEL OVERFLOW-TUNNEL.

5188.

Fig. 19.

COMPLETED BELLMOUTH OVERFLOW.

Fig. 20.

BELLMOUTH OVERFLOW IN USE.

Any leakage water from the reservoir is intercepted by the pits between the buttresses and cannot enter the sand-wedge, and an impermeable covering is provided to prevent access of the torrential rainfall which occurs every monsoon (*Fig. 8*).

Steps had to be taken during construction to maintain the supply of water drawn from the Shing Mun river, the invert-level of the channel, which was on the right bank of the stream, being about 485 O.D. where it passed through the dam.

This was effected by leaving a passage through the thrust-block which was afterwards filled with concrete and grouted, a culvert being formed through the rock-fill, which was subsequently made solid with concrete and " plums."

MOVEMENTS OF THE THRUST-BLOCK.

The rotation of the thrust-block about a horizontal axis was measured by means of a water-level, which consisted of a pipe 80 feet in length laid below the transverse gallery shown on Fig. 3, Plate 1, at level 453 O.D., that is, 93 feet above stream bed-level, the pipe being connected at each end to vertical tubes, the level of the water in the tubes being ascertained by micrometer point-gauges, it being found possible to measure the deflexion of the dam below that level to about $\frac{1}{100}$ inch. A plumb-line, which is housed in the 18-inch diameter vertical pipe, extending from road level to the transverse gallery (see Fig. 3, Plate 1) was also used when the thrust-block had been completed, to ascertain the deflexion of the upper portion of the dam, 182 feet in height.

Owing to the shortage of water in the Colony it was decided to impound water before the thrust-block had been completed, and the inlet to the tunnel was therefore closed at the end of the dry season in 1935, the reservoir filling to about level 500, a depth of about 140 feet.

When the reservoir was first filled it was anticipated that the effect of the water-pressure would be to cause the dam to deflect downstream, but the reverse proved to be the case as far as the lower portion of the dam was concerned. When, however, pendulum measurements commenced, it became evident that the upper portion of the dam, that is, above level 453 O.D., deflected downstream with increasing water-level.

It has not been possible to trace any exact relationship between the deflexion of the lower portion of the dam and the level of the water in the reservoir, although there is a general tendency for the deflexion upstream to decrease with decreased water-pressure.

Fig. 10 (p. 196) shows the deflexion from the vertical of the lower and upper portions of the dam in relation to the water-level in the reservoir.

The amount of deflexion of the lower portion is arrived at by assuming that no movement has taken place below rock-level, and the deflexion

given for the upper portion is in relation to the vertical at level 453 before
water was impounded.

It will be seen that there is a relationship between the water-level and
deflexion as far as the upper portion of the dam is concerned, the maximum

Fig. 10.
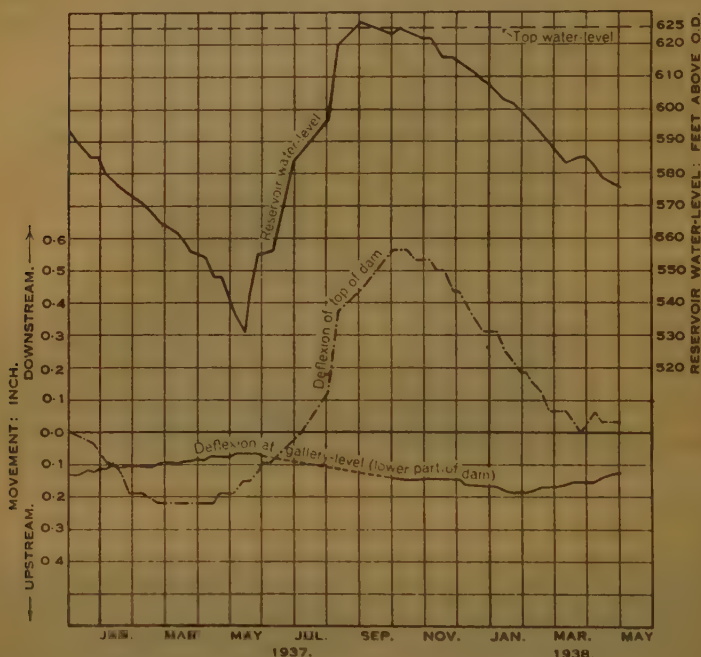
movement of the top in a downstream direction being 0·57 inch when the
reservoir is full, the maximum deflexion of the lower portion upstream
amounting to 0·18 inch.

DIVERSION-TUNNEL.

It was necessary to divert the river through a tunnel in order to permit
of the construction of the dam during the monsoon period, this tunnel
being also utilized for the conveyance of the supply and scour-pipes, and
for the disposal of flood-water.

The tunnel is 15 feet in internal diameter, lined with concrete containing
600 lb. of cement per cubic yard, having a minimum thickness of 12 inches,
the diameter being determined by the estimated maximum rate of discharge
of flood-water.   At every 5 feet along the tunnel five pipes were provided
passing through the tunnel lining spaced 5 feet apart, for the purpose of
injecting grout between the concrete in the crown and the rock.

The diversion-tunnel is shown by full lines on Figs. 11, Plate 1, the bellmouth-overflow and inclined tunnel (to be presently described), which joins it, being indicated by dotted lines.

A 48-inch scour-pipe is laid below the floor of the tunnel, being controlled by two valves, one in the tunnel and the other at the outlet, and is turned upwards at its inlet end, terminating by a bellmouth at level 410 O.D.

The scour-pipe is reduced to 30 inches in diameter at the outlet end and terminates in a jet-disperser, the scour- and supply-pipes being cross connected and provided with valves so as to permit water for supply being drawn through the scour-pipe.

A special valve was also fixed close to the tunnel inlet, by means of which the stream could be passed through the scour-pipe in dry weather during the placing of the stopping at the inlet to the tunnel (Figs. 12, Plate 2).

The supply-pipe, 48 inches in diameter, was placed below the tunnel invert for that length of the tunnel which would be used for the disposal of flood-water, being brought above invert-level between the stopping forming the bend at the junction of the inclined tunnel and the diversion-tunnel and the stand-pipe in the valve-tower after the inlet to the tunnel had been closed.

The valve-tower, which is 18 feet in internal diameter, is placed vertically over the diversion-tunnel as shown on Figs. 11, Plate 1, and the entrance to the tunnel was lined with cast-iron rings as far as the valve-shaft, the space between the metal and the rock being filled with concrete containing 600 lb. of cement per cubic yard.

The rock between the valve-shaft and the inlet was so jointed that this portion of the tunnel would be under a water-pressure amounting to 230 feet at the crown of the tunnel, and therefore the following precautions were taken to prevent leakage into the tunnel.

In order to ensure that the concrete completely filled the space above the cast-iron lining, a hole was left in the wall of the valve-tower which gave access to a recess in the rock formed by an extension upwards of the tunnel-excavation by about 5 feet, which was filled with concrete after grout had been injected at a pressure of 100 lb. per square inch, so as to fill any space left between the concrete placed inside the cast-iron lining to form the bulkhead and that lining.

Between the bulkhead and the valve-shaft, circumferential grooves were provided in the cast-iron lining for the insertion of a rubber ring (Figs. 13, Plate 2), and V-grooves were formed for the insertion of lead-wool caulking in both the longitudinal and circumferential joints between the segments. Conical lead grummets were also inserted in a space left for their reception below the bolt-washers, which were compressed as the joints were tightened up so as to prevent leakage.

BULKHEAD.

The mouth of the tunnel is closed by means of a cast-iron bulkhead con-
sisting of flanged plates, nine in number, which could be rapidly assembled
(Figs. 13, Plate 2). The joints between these plates were caulked in the
ordinary manner and the tunnel filled with concrete containing 600 lb.
of cement per cubic yard for a distance of 35 feet behind the cast-iron
bulkhead.

VALVE-TOWER.

The valve-tower is 18 feet in internal diameter (Figs. 14, Plate 2)
faced internally and externally with moulded concrete-blocks, the external
blocks being so shaped as to form an octagonal tower to above rock-level,
the excavation being taken out so as to provide a total thickness of 5 feet
for the lining below that level.

The inner concrete-blocks are of uniform thickness (12 inches) so as to
provide a smooth face from top to bottom for the application of an asphalt
joint $\frac{3}{4}$ inch in thickness, to cut off any water which might percolate
through the concrete under external water-pressure.

The walls increase in thickness at different levels, the minimum being
5 feet 6 inches down to level 572·50 O.D., 7 feet between 572·5 O.D. and
532·5 O.D., and 8 feet until the shaft enters the rock.

It was originally specified that the asphalt should be applied in two
layers to the back of the inner blocks, being brought up in advance of the
concrete, but owing to the difficulty of obtaining a surface sufficiently dry
for its adhesion it was finally decided to mould the asphalt in strips, the
joints being carefully made so as to form one continuous cylinder brought
up in advance of the concrete, any space between this ring and the blocks
being grouted.

The tower contains a 48-inch internal diameter standpipe, supported
on a cast-iron duckfoot-bend which is a vertical continuation of the 48-inch
supply-pipe, surmounted by a conical cap and an air release-pipe. Intake
pipes 30 inches in diameter, controlled by duplicate valves, discharge
into the standpipe at levels 479·75, 537·50, 576, and 614·5 O.D. (Figs. 14,
Plate 2), the lowest level being governed by the conduit into which the
supply-pipe discharges.

The valve-tower is surmounted by a house in which are placed two
water-level recorders and from which a stairway descends giving access
to the headstocks controlling the valves on the intakes.

Two float-pipes are provided for recording the water level, the float in
one of them operating a pointer indicating the level of the water in the
reservoir on a circular scale. The other float moves a pen, indicating on
a clock-driven drum-diagram the level of the water above top water-level
on an open scale (3 inches to 1 foot), so as to enable the depth of overflow
to be accurately measured.

Six pens are also provided for indicating on the diagram the period during which any one of the siphons, to be presently described, is in operation, the pens being electrically operated in the following manner. A copper ball 8 inches in diameter attached to one end of a lever is placed in the mouth of each siphon, which ball moves under the pressure exerted by the water when the siphon comes into operation, closing a circuit which causes the pen to make a rising trace on the paper. When the siphon ceases to flow the ball regains its original position, the circuit being thereby opened, allowing the pen to fall to its former position.

Access to the platforms on which the headstocks controlling the lower intakes are placed is obtained by a spiral staircase formed of reinforced concrete.

VALVE-TOWER BRIDGE.

Access to the valve-house is obtained by means of a lattice-girder steel bridge (Figs. 15, Plate 2) 73 feet 3 inches in length, supported at one end by means of a reinforced-concrete bracket extending outwards from the thrust-block, and at the other end on a bracket incorporated in the wall of the tower, the bridge-girders being anchored at the dam and free to move on roller-bearings at the other end.

The steelwork was galvanized, before being sent out by the manufacturers in six lengths, to be bolted together after arrival. These lengths were packed in crates to prevent damage in transit, and arrived in perfect condition ; the bridge, which weighed 16 tons, was erected in 3 days, after which it received a final coat of grey paint.

FLOOD-DISPOSAL WORKS.

The Jubilee reservoir, which has a catchment area of 3,000 acres, is situated in a region where very heavy rainfalls have occurred, as much as 8 inches having fallen in 2 hours, and an unusual flood, which reached an estimated maximum run-off at the rate of 5 inches per hour from the catchment area, had been recorded.

The normal flood-hydrograph allowed for 5 inches per hour peak rate of run-off and a total of 18 inches in 12 hours from the direct catchment of 3,000 acres and 0·4 inches per hour to which the capacity of the catchwaters is limited : the peak inflow of 17,400 cusecs would be reduced to 11,330 cusecs by the equalizing effect of the reservoir, which has a top water-area of about 5 per cent. of that of the direct catchment.

The design of the dam did not permit " overflow " and the steep mountain sides and height of the dam rendered the construction of a separate flood-discharge weir and channel prohibitive in cost.

It was therefore decided to adopt a circular bellmouth weir, discharging into the river diversion-tunnel, but owing to the configuration of the

ground it was not possible to place this weir directly over the tunnel, and the arrangement shown on Figs. 7 and 11, Plate 1, was therefore adopted, the discharge flowing over the weir and passing down an inclined tunnel 15 feet in diameter which joined the diversion-tunnel about 260 feet from its outlet.

The bellmouth-overflow was, of necessity, located on a steep slope (Fig. 16, Plate 2) which involved a large amount of excavation to form a channel of approach to all parts of the periphery of such dimension that the tangential velocity should not exceed 4 feet per second, with the consequence that the diameter of the bellmouth and length of weir were limited by economic considerations to 74 feet and 228 feet respectively.

The conditions being somewhat unusual, it was decided to undertake a series of model-tests, which are described in detail elsewhere [1], and will therefore be only briefly referred to.

These tests were carried out with models having the following scale ratios : 1 to 19, 1 to 24, 1 to 29·4, and 1 to 43·5, with the object of determining :—

(a) The best shape of the bellmouth.
(b) The relation between discharge and depth of overflow.
(c) The vacua set up under high velocities of discharge round the bend where the shaft joins the inclined tunnel and in the tunnel.

Previous tests carried out at the Burnhope reservoir, which are also described elsewhere [2], had dealt with the best method of preventing the formation of a major vortex which would produce unstable conditions, reducing the discharging capacity of the weir, and the amount of air which would be drawn down with the water at various rates of discharge.

The result of these experiments showed that the best way of preventing the formation of a major vortex was to divide the circular weir into two parts by means of a curtain-wall extending above the highest level which the water would reach, the bottom of this wall being 3 feet below the lip of the weir. This wall is continued across the approach-channel so as to form two semicircular weirs with separate approach-channels (Fig. 7, Plate 1 and Fig. 16, Plate 2). Large quantities of air are drawn down with the water at low rates of discharge, but the amount of air gradually decreases until the tunnel becomes gorged, when the volume amounts to about 25 per cent. of the discharge over the lip of the bellmouth.

The Burnhope model-tunnel was made of transparent celluloid so that observation of the interior was possible, and it will be seen from *Fig. 17* (facing p. 194) that the air rises immediately to the roof of the tunnel,

[1] G. M. Binnie, " Model-Experiments on Bellmouth and Siphon-Bellmouth Overflow Spillways." Journal Inst. C.E., vol. 10 (1938–39), p. 65. (November 1938.)
[2] W. J. E. Binnie, " Bellmouth Weirs and Tunnel Outlets for the disposal of Flood Water." Trans. Inst. W.E., vol. xlii (1937), p. 103.

and when the actual discharge of the model was compared with the theoretical discharge, the air appeared to have no effect on the discharging capacity if the tunnel was assumed to be running full.

The Burnhope experiments had shown that considerable vacua were set up at the roof of the bend where the shaft joined the tunnel, and it was therefore decided to introduce a longitudinal duct in the roof of the Gorge dam-tunnel with an air-pipe 24 inches in diameter to draw air from the surface to prevent the formation of any considerable vacua (Fig. 18, Plate 2).

The model tests carried out in connexion with the Jubilee reservoir showed :—

   (a) That the original design of the bellmouth, which was similar to those previously constructed, required modification owing to oscillations of water-level which were set up when the discharge exceeded 5,000 cusecs due to the formation and destruction of a partial vacuum whether the air vents were opened or closed. After various modifications were made, the shape of the bellmouth and shaft, illustrated in Fig. 18, Plate 2, was evolved, which gave a steady flow up to 10,500 cusecs.

   (b) The value of C in the formula $Q = LCH^{\frac{3}{2}}$, where H denotes the depth of overflow and L the effective length of the weir (228 feet), was approximately 3·14 up to $H = 7$ feet, giving a value for Q of 13,000 cusecs, beyond which the coefficient decreased, the maximum discharge being 15,000 cusecs when $H = 8$ feet.

   (c) The tests carried out with the models to various scales showed that no vacua were set up until a discharge of about 8,500 cusecs was reached with the modified design, whether the air vents were opened or closed.

   (d) That the vacua reached a certain value, after which there was a decline.

Siphons.

As a result of these experiments it was decided to incorporate six automatic siphons in the dam, each capable of discharging 500 cusecs, three of which were designed to commence to prime at 625·05 O.D. and three when the depth of overflow had reached 626 feet, the top water-level being 625 O.D.

Assuming a maximum outflow of 11,330 cusecs, the depth of overflow would be 5 feet leaving a freeboard of 5 feet, and the discharge over the bellmouth 8,330 cusecs, which is well within the limit of "steady" flow indicated by the models, the velocity in the tunnel being 47·1 feet per second. An outflow of 11,330 cusecs is equivalent to 3·7 inches per hour running off the catchment area, and the maximum discharging capacity

of the bellmouth and siphons, as predicted by model-tests, amounts to 16,000 cusecs, allowing for 3 feet of freeboard, which is equivalent to approximately 5·25 inches per hour.

Owing to the long intervals of time which may elapse before the occurrence of a major flood, or the lack of means of measurement, the Authors have been unable to find any published records showing how the discharge inferred from models of bellmouth weirs agreed with the actual quantity of water passing into the river below during a flood of maximum intensity.

The model-experiments carried out for the Jubilee reservoir indicated some slight improvement in the coefficient as the scale-ratio increased, but as the largest model was only a little over twice the size of the smallest, the data collected were not conclusive.

The Burnhope reservoir (Durham County Water Board) is provided with an overflow-weir over which the water passes before entering the bell mouth and another weir below the tunnel-outlet, the depth of overflow being continuously recorded for the bellmouth and both weirs.

The maximum "outflow" in this case was calculated to amount to 2,630 cusecs, and a flood occurred in October 1936 which amounted to 1,300 cusecs, computed from the model experiments, as compared with a discharge of 1,390 cusecs the mean of the computation of the discharge over the other two weirs.

It is therefore probable that model-tests on bellmouth-spillways under-estimate the discharge at corresponding depths of overflow which will be obtained with the prototype.

DESCRIPTION OF BELLMOUTH-OVERFLOW.

The bellmouth is constructed of concrete containing 600 lb. of cement per cubic yard, the thickness of the wall being determined by the shape of the structure and the weight necessary to prevent flotation. It is faced internally with small granite blocks, as shown in *Fig. 19* (facing p. 195), which extend round the bend as far as the straight portion of the inclined tunnel. A benching was excavated to form the approach-channel to the landward portion of the weir, the lip of which is 5 feet above floor-level.

The ground fell so steeply on the reservoir side that the bellmouth assumed a stepped "mushroom" shape (Fig. 18, Plate 2), the concrete being brought up in steps 3 feet in height cast in situ, false joints being introduced to imitate masonry blocks, and steel rings 12 inches deep and ⅜-inch in thickness were inserted at the top of each step and incorporated in the one above so as to reinforce the structure and to prevent creep of water between successive steps.

The crest of the weir is formed of granite blocks and the curtain-wall, which is supported by the weir-crest, is of reinforced concrete, 2 feet in thickness, faced with small granite blocks.

The top of this wall is 10 feet above overflow level and is splayed out to form a footway, provided with handrails, so that inspection of the interior of the bellmouth is possible and repairs facilitated.

The wall is fixed at the landward end but rests on a steel plate covered with bituminous sheeting at the other end, so as to be free to move in a groove, 3 feet in depth, formed in the crest of the weir, in order to prevent stresses being set up in the bellmouth by variations in the length of the wall due to changes of temperature, the vertical joints separating the wall from the granite crest-blocks being sealed with bituminous sheeting and bitumen to prevent leakage.

Overflow to a depth of about 1 foot 10 inches took place in September, 1937, and afforded a magnificent spectacle when viewed from the footway (*Fig. 20*, facing p. 195).

DESCRIPTION OF SIPHONS.

The siphons, which are six in number, calculated to discharge 3,000 cusecs, are incorporated in the solid portion of the thrust-block at the

Fig. 21.

SECTION THROUGH SIPHONS.
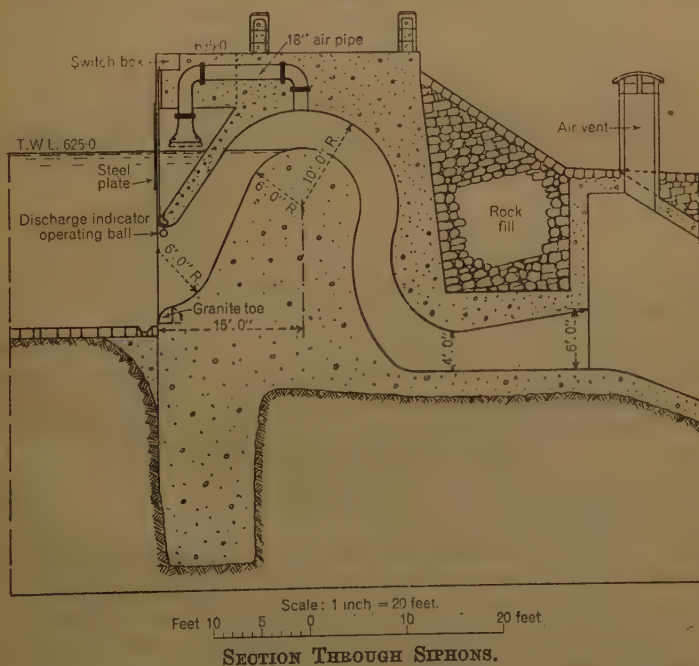
northern end of the dam, a section through one of the siphons being shown on *Figs. 21* and *22*.

The crest, which has a radius of 6 feet, is 6 feet long, the throat having

a depth of 4 feet, so that the throat area amounts to 24 square feet, the discharge "leg" of the siphon being 23 feet measured from crest to invert level at the outlet, the effective suction head being approximately 21 feet.

The "hood" is incorporated in the dam, the mouth having an area of 48 square feet, the top being submerged 8 feet below overflow-level, and forms the bottom of a "stilling" basin to which water is admitted from the reservoir through a slot 4 feet below water-level, formed by a steel plate bolted to the sidewalls separating the siphons.

The cast-iron air pipe, 18 inches in diameter, is provided with a horizontal bellmouth so placed as to become immersed when the water has risen to a predetermined level in the "stilling" basin, when priming commences.

Fig. 22.

SECTION THROUGH SIPHON TUNNEL.
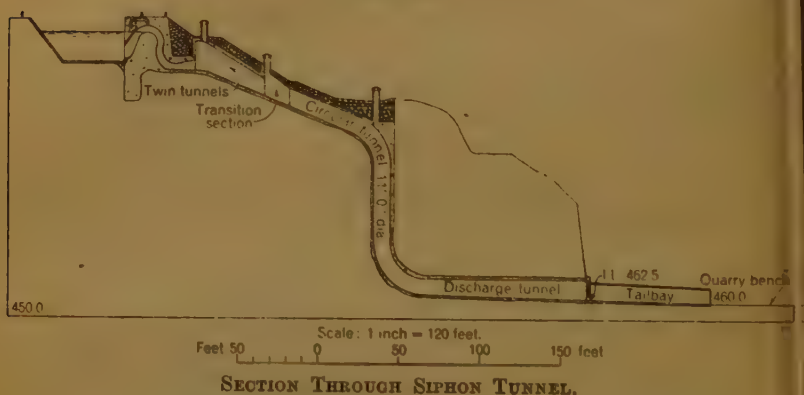
The air-pipe communicates with the crown of the siphon and is built into a cross wall, so as to secure the pipe against vibration.

Gulping, due to wave action, is minimized by means of a reinforced-concrete curtain-wall across the entrance to a basin excavated in front of the siphons to a depth of 2 feet below the level of the mouth, in order to admit water from the reservoir.

This wall extends 4 feet 6 inches below overflow-level, the top being carried up 6 feet above that level, the bottom being supported on pillars resting on the benching.

The "leg" of the siphon is S-shaped to facilitate priming, and is bent round so as to discharge horizontally into a chamber with a semicircular roof of sufficient width to accommodate three siphons, a vent shaft 3 feet in diameter being provided immediately above the point of discharge.

This chamber has a width of 22 feet at the point of discharge which gradually decreases to 9 feet 10 inches where the two chambers converge to enter another semicircular chamber 21 feet 9½ inches in width and

7 feet in height, which converges to form a circular outlet 11 feet in dia-meter, and another vent-shaft is provided at the entry.

The discharge-tunnel bends round on a vertical curve having a radius of 23 feet, the water entering a shaft 80 feet in depth which bends round at the bottom to discharge the water horizontally on to a benching at level 460 O.D. which was formed during the removal of rock from the quarry which furnished stone for the dam.

A wall is provided near the edge of the benching which forms a channel conveying the water discharged from the siphons to a point below the dam, where it falls over a cliff into the valley below, the drop being about 100 feet.

This somewhat complicated design to provide for the discharge from the siphons was rendered necessary in order to convey the water below the rock-fill.

Experiments were carried out with models of siphons of various shapes to determine the depth at which the siphon would prime, and the effect of providing the outlet with a water-seal or allowing the water to discharge horizontally, as the only permissible design of the outlet-chamber was such as to render it impracticable to provide a water-seal.

The result of the experiments showed that the siphons primed at practically the same depth on the crest whether a water-seal was provided at the outlet or not.

A "frame" and travelling carriage to hold a current-meter has been provided, arranged so that the velocity of the water entering the mouth of a siphon can be measured at any point in order to check the calculated discharge, but so far no records have been obtained.

Conclusion and Acknowledgements.

The whole of the constructional work was substantially completed towards the end of 1936 ; it was not until after the middle of 1933 that the final location and general features of the design of the Gorge dam were decided upon, and that the work was completed in 2½ years' active con-struction is a tribute to the executive skill and organizing capacity of Mr. Hull and to the loyal service of his staff.

The major items involved in the Gorge dam were :—

Excavation and concrete in cut-off trench	20,000	cubic yards.
Concrete in diaphragm	9,000	,,
Concrete in thrust-block	130,000	,,
Rock-fill	480,000	,,
Sand-wedge	27,000	,,

The cement used was manufactured by the Green Island Cement Co., Kowloon, and was transported by means of special containers holding 5 tons in barges to the point nearest the commencement of the access road, and thence by lorry, the containers being emptied into silos on arrival at the site.

All the plant used was of British manufacture and this, with pipes, valves, etc., was purchased and shipped by the Crown Agents for the Colonies.

Finally, the Authors would desire to record their appreciation of the co-operation so willingly afforded by the Public Works Department, and, in particular, of the personal interest taken in the scheme by the Director.

The Paper is accompanied by twelve sheets of drawings and four photographs, from which Plates 1 and 2, the Figures in the text, and the half-tone page-plate have been prepared.

Discussion.

Mr. H. J. F. Gourley, in presenting the Paper, showed a number of lantern-slides illustrating the works described.

Sir Clement Hindley, Vice-President, in proposing a vote of thanks to the Authors for their Paper, said the present occasion appeared to be unique, as it was, he believed, the first time in the history of The Institution that a Paper had been presented by the President during his term of office. The Paper related to a work of very great magnitude and of outstanding importance in the locality where it had been erected. The work possessed some very unusual and interesting features, particularly the measures which had been taken to prevent progressive percolation. It was recognized that progressive percolation through the concrete could be a most serious danger, and the work done in the present instance, particularly in relation to the cut-off curtain and the diaphragm, showed great ingenuity and a full appreciation of the dangers which had to be met. Although about 6 years elapsed from the time of the beginning of the survey to the time of completion, it was of interest to find that the work itself was carried out in a period of $2\frac{1}{2}$ years, a most remarkable feat under the conditions in which it had to be done. It would be of interest to the members, as it always was in such cases, to know the total cost of the work.

The President, in inviting Sir William Peel to take part in the Discussion, said that Sir William was Governor of Hong Kong at the time that the works were in progress.

Sir William Peel said the history of the inception of the scheme was better known to Mr. H. T. Creasy, who was Director of Public Works when Sir William went to Hong Kong in 1930, than to himself. The scheme had been under discussion for some time, but some difficulty had been experienced in obtaining the necessary authorization to proceed with the work. When he came home on leave in 1932, however, he got the Colonial Office to cable to Hong Kong to say that the preliminary work might start, and to inform the Engineers at home that they could make their preparations and get on with the work; later in the year, he travelled out with Mr. G. B. G. Hull, the resident engineer, who put the work in hand at once.

He would like to congratulate the Authors and all those concerned on the work that they did in connexion with malaria, which threatened them from the very beginning of their work. At Shing Mun merely to oil all the pools in the depressions formed by the earthworks would not have met the difficulty at all adequately. It was a very bad malaria area, and miles of anti-malaria drains had to be constructed in a big circle around the

Shing Mun dam. There could be no doubt that that saved the situation because at the beginning the mortality was heavy amongst the Chinese labour; although malaria was never eradicated entirely, the anti-malaria drains, which were well constructed, reduced the sickness and mortality to a minimum.

Mr. W. T. Halcrow remarked that the dam consisted of a watertight skin on the upstream face of an embankment which provided the necessary weight to resist the thrust of the water. It thus differed materially from the usual form of embankment dam, where watertightness was obtained by a central core of clay or concrete. The principle governing the design was one to which some attention had been given in recent years, and Mr. Halcrow believed that in America some small dams had been built consisting of a stone bank with a sheet of steel, or an equivalent seal, on the upstream face. Where the estimated life of a dam was short, as in the case of certain mining works, that design was no doubt economical, but such a simple design could not be adopted for the Gorge dam, where an indefinite life was required. It had occurred to him, on looking at the cross section of the Gorge dam, that the thrust-block, which was a very heavy mass of concrete, might have been to some extent, if not entirely, dispensed with had the upstream slope been made flatter. With a stone bank, the chief difficulty was to find a suitable design or material for a flexible watertight face to allow for movement of the supporting bank under the weight of the water. It was possible that the articulated slabs of the Gorge dam, which the Authors had designed with great care, might be adapted in such a case.

Some years ago in Norway a concrete dam, which was leaking badly and was rapidly becoming unsafe, had been repaired by building a diaphragm-wall some 3 or 4 feet in front of the dam and supporting it by struts from the face. The diaphragm had been made watertight, and the dam provided the supporting weight without alteration. Any seepage through the diaphragm wall was drained from the bottom of the space between the diaphragm and the dam. That design could be adopted in the case of a dam to be constructed. Only recently he had had to examine a similar repair scheme for another dam in Norway, which was built of masonry and leaked badly. A watertight facing was to be provided, with drainage, leaving the existing dam untouched. In those cases the diaphragm did not require to be articulated, as the supporting wall would not settle. The principle upon which the design was based was, however, the same in each instance.

Whilst he agreed generally with the remarks about the incorporation of a rich and poor concrete deposited at the same time, which was customary in the construction of concrete dams, he had never come across any cases where visual cracks had been located between the two classes of concrete, but in his own work he had endeavoured as far as possible to reduce to a minimum the thickness of the rich concrete. It was difficult

to secure a watertight junction between concrete which had already set and green concrete deposited above it, and he liked the method adopted in the present instance. The introduction of copper strips between the lifts was a positive way of ensuring that there should be no seepage, and he was surprised that it had not been more generally adopted. He had some doubts regarding the placing of concrete in 20-foot lifts. Very strong shuttering was required, and unless girders of great depth and stiffness were used there was bound to be some deflexion as the weight was imposed. There was no certainty that the concrete in the bottom part of the lift had not set to some extent before all the concrete had been deposited, and that might affect the strength or impermeability. With regard to the " Callen-drite " sheeting which had been used extensively, he would like to ask the Authors if they considered that it would have an indefinite life, and, if not, whether in the course of a few years it would have fulfilled its purpose and would not be required to function.

The reason given for the adoption of rock-fill instead of earth-fill to support the thrust-block was that the water might find its way into the earth embankment and cause slips. In view of the provision of the drainage-space between the diaphragm-wall and the thrust-block, there would not appear to be much risk of percolating water affecting the sup-ported embankment. However, Chinese labour was cheap and there was no doubt that if the extra cost of stone over earth-fill was small, the stone was much to be preferred.

The sand-wedge was interesting, and its function was explained. He would be a little doubtful about depending upon it for the stability of the upper part of the thrust-block. Whilst precautions had been taken to ensure that the sand would not find its way into the rock-fill by having the face of it set in cement mortar, it was mentioned in the Paper that one of the considerations borne in mind in the design was the possibility of earth-tremors, and it appeared to him that in the event of earth-tremors occur-ring of sufficient strength to cause substantial fissures in the face there might be risk of sand escaping and taking away the support of the thrust-wall. In his view, the stone backing could be carefully placed behind the thrust-block or consolidated by a heavy roller so as to ensure a solid bearing of the block on the stone.

The description of the movements of the thrust-block was extremely interesting, as it was not usual to find a dam tilting upstream when the water-pressure was applied. The following might be a possible explana-tion. The rock on which the dam was built contained cracks and joints, some of which were filled with clay or china clay. When the reservoir was full, the floor of the reservoir immediately upstream of the dam was com-pressed slightly, and so caused an upward tilting of the thrust-block. Movements of structures such as jetties were known to occur when the tide rose and fell, a jetty sinking as the weight of water on the foundation was increased and rising when the tide ebbed.

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Mr. H. T. Creasy remarked that the island of Hong Kong was studded
with reservoirs, those reservoirs being connected with catchwaters that
prevented the greater part of the water running from the hills down into
the sea, which took place very quickly, the island being a mass of high-
peaked hills.

The crisis in regard to water-supply came about in 1929, when the
reservoirs were depleted and there was no means of getting water across
from the mainland except in water-boats which were requisitioned. The
position was such at that time that ships coming to Hong Kong were asked
to fill up their tanks, so as to bring Hong Kong water. The total capacity
of the reservoirs on the island was between 10,000,000 and 11,000,000
gallons per day, which was really the requirement of the island, so that
anything else had to be brought across from the mainland by means of a
pipe-line that had been laid to connect with the Kowloon reservoirs, which
were then giving 5,000,000 gallons per day. The pipe which was first put
across was able to supply to the island 3,000,000 gallons per day.
Fortunately, after the arrival of Sir William Peel as Governor the Shing
Mun scheme was sanctioned. The pipe referred to had since been
duplicated by a 24-inch diameter pipe.

Mr. J. S. Wilson observed that in earlier days a dam consisted of a
solid mass of concrete, which supplied the necessary weight and had an
impervious face, and that all the experiments with regard to stresses in
dams had been related to the stresses in such a homogeneous structure.
Since then there had been great changes, and the idea of a homogeneous
mass had been abandoned; in fact, it was stated in the Paper that the
enormous natural granite beds in the vicinity were divided up into com-
paratively small blocks by fissures. Not long ago some very careful
experiments had been carried out in America in an attempt to measure the
stresses in a concrete dam, but although great precautions had been taken
to obtain accurate figures, no exact results had been obtained. The
intention had been to measure the actual strains produced in the concrete
by the pressure of the water, but it was found that those strains were
rendered invisible by temperature-strains, creep-strains, and so on. In the
Gorge dam the various requirements were met individually. The
penetration of the water was entirely prevented by the face, which was
capable of conforming to all the local movements which might take place.
To offer resistance to the face there was the thrust-block, which trans-
mitted the thrust in a uniform manner to the large mass of packed granite.
In addition to those provisions, the Authors had had to contemplate the
possibility of earthquakes.

The deflexion diagram (*Fig. 10*, p. 196) was very interesting, and he was
sorry that the Authors had not given their interpretations of the meaning
of the curves. Had the Authors carried out any investigations into the
stress–strain relation of the rock-fill?

Mr. G. M. Binnie observed that, as stated in the Paper, the diaphragm

was poured to a height of 20 feet in one operation. Before shuttering could be designed for such a high lift, it was necessary to carry out a full-scale experiment to determine the pressures likely to be exerted by the concrete during placing, and he showed a lantern-slide illustrating the apparatus used.

Two concrete side walls 21 feet high were built against a vertical wall with a space 2 feet wide between them. The remaining side consisted of planks 2 feet long, set at a batter of 3·4 to 1 for the lower two-thirds and vertically for the upper one-third of the total height, thus corresponding to the face of the diaphragm. The ends of the planks rested on steel bars placed horizontally across knife-edges built into small openings left in the side walls. The concrete was poured into the form thus made in one operation in 3½ hours, giving an average rate of rise of about 6 feet per hour. The pressures against the planks were transferred at the ends on to the steel bars, thus causing the bars to deflect under the load. The pressures exerted by the concrete were deduced from the deflexion of those simply-supported steel bars, the deflexions being measured relative to a second row of rigid bars by means of a micrometer.

The concrete used was a good workable mix containing 600 lb. of cement per cubic yard, with an average slump of 1 inch. Two men were engaged in placing the concrete, one with a spade and the other with an automatic rammer. The results indicated that the pressure against the shuttering at a batter of 3·4 to 1 was equivalent to that of a fluid with a density of 110 lb. per cubic foot, up to a maximum of 330 lb. per square foot at 3 feet head, no increase in pressure being observed above that head. A similar experiment was also carried out for a vertical face 16 feet high in one pouring. The results of the latter experiment indicated that the pressure against vertical shuttering was equivalent to that of a fluid with a density of 124 lb. per cubic foot, up to a maximum of 558 lb. per square foot at 4½ feet head, without any increase in pressure above that head.

The results obtained from those experiments were used as the basis for the design of the steel shuttering, the working stress being 8 tons per square inch. Mr. Binnie showed a model of a steel panel, and said that the panels as finally constructed weighed 1,200 lb. each, seven panels making up the width of 25 feet. The panels were each used about twenty times during the construction period, and were in excellent condition at the end of the job. They showed no signs of fatigue, indicating that the concrete pressures deduced from the experiments were evidently safe assumptions.

In the Paper it was stated that the steel panels were held at the bottom by means of a groove in the step below, and at the top by tie-bars fixed to the thrust-block. They were thus supported with a 20-foot clear span, without any bolts or wires liable to rust and cause leakage going into or through the concrete.

The panels deflected slightly under the load. When the upper ends were released after the concrete had set, the panels sprang away approxi-

mately ½ inch at the top, thus automatically stripping themselves from the concrete. As there were no bolts from the concrete projecting through the panels, the panels could then be lifted immediately and placed elsewhere wherever required. Owing to the ease with which the panels could be erected and stripped and the great number of times each panel was used, the cost of shuttering per square yard for the steel panels compared very favourably with the usual wooden shuttering.

One of the principal advantages of the high-lift construction was that it gave only one horizontal joint every 20 feet to be made good against leakages, instead of from four to six joints with the more usual lifts. In spite of the exceptionally high lift poured in one operation, the average rise of temperature of the diaphragm-concrete, containing 690 lb. of cement per cubic yard, was only 38° F., as compared with 53° F. for the concrete containing 600 lb. of cement per cubic yard in the tongue trench. That moderate rise in temperature for so rich a mix could be accounted for by the large surface area exposed for dissipation of the heat generated by the setting concrete. The high lift had also the effect of compressing the semi-liquid mass under its own weight during construction, so that a very dense concrete was obtained, which proved to be absolutely watertight. The following were some of the advantages of the high-lift type of construction: complete absence of steel connexions into or through the concrete capable of rusting and permitting leakage; ease and economy of construction; reduction in the number of horizontal joints in a given height, giving fewer possible paths for leakage; and improved density of the concrete.

Mr. J. M. B. Stuart said that in various places in the Paper it was mentioned that the underlying material was granite, but it was not stated definitely that the catchment was on granite. In other parts where granite catchments had been found the water coming off those catchments was soft and had a progressive disintegration-effect upon the concrete in dams. In the Gorge dam, there seemed to be no doubt that there was considerable water-pressure under the concrete in the base of the thrust-block, as evidenced by the leakage into the inspection-gallery, and if the water were soft there might be some danger of deterioration in the concrete at the base of the thrust-block. In connexion with the stability of the dam, the thrust-block was founded on sound rock but it had not been taken in very deeply, and there was no cut-off; there therefore seemed to be no doubt that there would be a certain amount of leakage under the thrust-block. It was not thought necessary to remove the sand and boulders which overlaid the solid rock in the stream-bed from the area where the rock-fill was going to be deposited. The compression of the material in the bed of the gorge underneath the rock-fill was about 4 feet. The rainfall in the locality seemed to be very heavy at times, and the rainfall would certainly percolate through the rock-fill and get down to the foundations of the dam. The water percolating along at the founda-

tions of the dam under the rock-fill might possibly cause further settlement of the rock-fill. The rock-fill exerted a certain amount of passive resistance to the water-pressure, but with further settlement of it there might be some downstream movement of the thrust-block. The sand-wedge was apparently designed to counter that, but would the sand-wedge be active enough to have the desired effect ?

He was doubtful if the provisions made for flood-disposal were sufficient. It was stated on p. 199 that the actual run-off in the past from the catchment had been at the rate of 5 inches per hour, and that the flood-hydrograph showed a peak inflow of 17,400 cusecs. On p. 202 it was stated that the maximum discharging capacity of the bellmouth and siphons amounted to 16,000 cusecs, allowing 3 feet of freeboard, which was equivalent to approximately 5·25 inches per hour. If there had been a run-off of 5 inches in the past, to make provision for 5·25 inches in a work of the kind in question did not seem quite sound, nor did he think that that provision would agree with what was recommended in the Interim Report of the Committee of The Institution on Floods in Relation to Reservoir Practice [1]. The provision made for flood-disposal might be quite sufficient for normal floods, but it seemed to him that it might not be anything like sufficient for the really big flood which occurred once in 50 or 100 years and upset all previous calculations. The flood-discharge system consisted of the bellmouth and the siphons, and they had very little overload capacity ; there was a freeboard of only 3 feet with a discharge of 16,000 cusecs. In a dam of the kind in question it was not possible to allow any overflow to take place, because it would mean the immediate destruction of the dam. The Pineapple Pass dam was referred to in the Paper : it closed up one of the outlets from the catchment. The actual level of that dam did not appear to be given, but it might be possible to construct an emergency spillway for a catastrophic flood so as to avoid any risk to the Gorge dam.

Mr. F. C. Temple, referring to the statements in the Paper that one of the considerations on which the design of the dam was based was possible earth-tremors, and that it was seen in Japan that the effect of an earthquake on a solid masonry dam might be serious, said that, in view of those statements, he was surprised that anything so solid as the cut-off wall and thrust-block should have been adopted. It might be of interest to give an illustration of how extraordinarily mobile the earth was during a big earthquake. The Bihar earthquake of 1934 extended over an area roughly 300 miles long and 100 miles wide ; in that area, of every four houses one was destroyed, two were more or less damaged, and one remained intact. Over four hundred bridges were down or unsafe, and nearly every earth embankment had subsided, whilst the sides of the rivers came in and their beds came up. That had produced the most extraordinary effects. In a bridge with two 100-foot spans, the centre well had remained where

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[1] Inst. C.E., 1933.

it was, and the two others had moved respectively 6 feet and 8 feet towards the centre, with the result that the girders had pushed the tops of the abutments over. The wells, however, were undamaged and practically vertical, and he had new abutments built on them. Everywhere there was a tendency to level things up. There were not very many reinforced-concrete structures in the area of the earthquake, but those that existed were all undamaged. A reinforced-concrete 50,000-gallon tank on a brick tower of piers and arches 40 feet high was undamaged, although the tower was split by a fissure 2 feet wide and had to come down.

For that sort of country it was obviously best to build the lightest practicable framed structures, insulated as far as possible from ground-tremors by a loose cut-off, of which the ideal pattern would be round balls in a tray. Actually there was a patent extant in New Zealand in which the superstructure did sit on ball bearings, although that form of con-struction was obviously impossible for a dam. Weight did not matter on so good a rock foundation, so that it was not likely that the Gorge dam would subside. If a big earthquake occurred, with the movement up and down the valley when the reservoir was full, the rock-fill at the back would be shaken down a good deal, and that would let the sand go. The water might sweep right up the valley, leaving half the dam bare. That type of movement had happened in the case of the Ganges, where the water left the south bank, travelled over to the north bank, exposing half the bed of the river, and then came back to the south bank to a height of some 20 feet above its proper level. The water in the dam, having gone up the valley, might come back with a rush and go right over the top into the loosened rock fill. Judging from the behaviour of structures which he saw in Bihar, he imagined that the most suitable form of dam for earth-quake country would be a multiple-arch dam of reinforced concrete, and he would like to know why the Authors had not adopted that design.

Mr. J. R. Davidson remarked that, in his opinion, the most notable feature of the design was the articulated diaphragm, or watertight face.

He understood that the term " panel ", as used in the Paper, referred to the vertical strip running the whole height above what was called the solid-face concrete ; namely, the whole of the vertical strip 25 feet in width, and not to the 20-foot lifts in which the panels were brought up. If that were so, it would seem that the panels were not attached in any rigid way to the thrust-block, but merely reclined against it, so that the main articulation was therefore on vertical lines between the adjoining panels, with the possibility of slight movement about a horizontal axis between each of the 20-foot lifts. The use of central vertical grooves filled with molten asphalt, and of bitumen sheeting placed on the flat surfaces on each side of each panel, was very interesting ; he had found that form of construction to be suitable for retaining-walls for reservoirs and filter-beds, but the method which had been adopted of forming the groove by means of a pre-cast concrete block was entirely new to him, and was a

particularly neat method. How were those pre-cast blocks held in position while the concrete was being placed ?

In the Authors' design, the " Callendrite " sheeting did not appear to enter the diamond-shaped groove. He had found it very advantageous to carry the bitumen sheeting a short distance into the groove, so that when the molten bitumen or asphalt was poured in, it partially melted the bitumen on the sheeting and got a grip on it. It was of great importance that the pouring of the asphalt should not be done until the concrete was thoroughly dry, and the groove should be kept free of water. For that purpose it was very convenient to build in a small horizontal pipe from the bottom of the groove, which acted as a drain, and which could be plugged immediately before the joint was run.

Reference had been made in the discussion to the question of the probable life of the " Callendrite " sheeting and the thin copper strips. He presumed that the " Callendrite " sheeting contained a foundation of some fabric, and he had found that for some curious reason fabric appeared to have the property of very slowly absorbing bitumen or similar substances ; in the present case, where the sheeting between the diaphragm and the buttresses was subject to very heavy pressure, he doubted if the sheeting would have a prolonged life. With regard to the thin copper sheeting, it was well known that in the case of several of the non-ferrous metals, such as copper and lead, with repeated bendings in opposite directions there was a hardening effect which tended to make the metal brittle, and it was then liable to break off. It was quite likely, however, that the movement of the copper in the case in question would be so slight that that effect would not be found.

Another point, also mentioned by Mr. Halcrow, was the placing of a rich mixture of concrete on the water-face of the block of concrete which was in contact with water. It was undoubtedly very valuable in providing an impermeable surface, and it had a much greater resistance to the action of water, but the Paper did not state the thickness of that rich mix. A joint of greased paper was put between the rich mix and the leaner mix. He had quite successfully put a rich facing of concrete on to the bulk of a leaner mix without any ill effect due to the difference in temperature. Mr. Davidson suggested that, since the resistance to water was very important, it would be very useful if the Building Research Station would consider making tests to ascertain what thickness of a rich facing of concrete could be put on to bulk concrete without causing trouble through difference in the rise in temperature.

*** **Mr. J. K. Hunter** was particularly interested in the description of the exploratory work carried out in the vicinity of the dam preparatory to making a final selection of the site. The work was apparently carried

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out by the orthodox methods of opening up trenches and sinking bore-holes, which were tedious and costly operations when a large area had to be covered, and the sound rock lay far below the surface. It appeared that after the site of the dam had been provisionally chosen, further borings disclosed that an extensive zone of decomposed granite covered a part of the site at a depth of 50 feet, and as a result of that discovery it was decided to change the location of the dam. During the past few years the practice of sub-surface exploration by electrical methods had been successfully developed, and had in several instances been used effectively in carrying out preliminary examinations of the sites for proposed engineering works, including dams. Such methods were rapid and appeared to be reasonably reliable ; moreover, they could be carried out at a small fraction of the cost entailed by boring. They did not supersede boring operations entirely, but by carrying out a comprehensive preliminary survey the most promising location could be chosen by examination of the rock contours so obtained, and core-borings might then be used to confirm the results of the electrical survey.

A few years ago he had been responsible for examining the site of a dam which it was proposed to locate on a granite formation. Although it was known that the granite was decomposed at the surface, it was hoped to obtain satisfactory foundations at a reasonable depth. Boring operations on the centre-line selected, however, revealed that in places total decomposition had extended to a depth of over 100 feet, and, although circumstances at the time prevented the scheme being proceeded with, it was considered probable that a more favourable site could have been found by altering the location. The work involved in those operations was slow and expensive, and, if the same or a similar site had to be investigated again, he would recommend that before any boring operations were put in hand a sub-surface examination should be attempted by one of the several methods of geophysical exploration which were now available.

Mr. B. D. Richards observed that the interesting and novel section adopted by the Authors was stated to have been dictated by considerations of economy and the avoidance of possible cracks, arising from either temperature-stresses or earth-tremors. It would be of interest to know whether or not the various alternatives considered included a reinforced-concrete arch dam, with wide abutments to distribute the pressure, and if so, how that compared in regard to cost. The diaphragm consisted of what might be described as a series of large concrete tiles, resting against, but not attached to, the buttresses ; the joints between the tiles, both vertical and horizontal, were staunched with copper strips. The tiles were thus practically held in place only by their weight. In the event of a serious earth-tremor passing up the valley at a time when the reservoir was partly empty, there would appear to be a tendency for a considerable stress to be put on the horizontal copper strips, and even for the tiles to be shaken off. That tendency would be much reduced by giving a flatter

batter to the upstream face or by some system of interlocking of the tiles. Presumably, however, earth-tremors of such magnitude were not anticipated.

In the plan (*Fig. 2*, p. 183) two gaps were shown on the western watershed, one of which was closed by the Pineapple Pass dam. Those gaps appeared to be possible sites for open weirs as an alternative to the escape tunnel and siphon-pipes, although the outlet tunnel would still be necessary. It would be of interest to learn whether or not such an alternative would be possible and economical.

He would like to know whether or not in making the model-experiments on the bellmouth overflow, described in detail in another Paper [1], the effect of placing an inverted cone on the vertical axis of the bellmouth had been tried, as a means of checking vortex-action, which was actually effected by placing a baffle across the diameter of the outlet.

Mr. R. C. S. Walters was concerned with building dams on granite in Cornwall. Cornish granite consisted broadly of quartz, orthoclase felspar, black biotite mica, and white muscovite mica ; it was less acid than the Burrator (Dartmoor) variety. The river valleys were sometimes wide, forming excellent storage-areas, and at other places were almost as narrow as gorges, forming superficially good sites for dams, and apparently resembling the conditions of the Shing Mun, but on a smaller scale.

Although granite was the formation for the whole of the valley, its composition or physical resistance to denudation obviously varied, but the reasons did not seem to be known. Disintegrated granite generally occurred at variable depths, such as at 105 feet below the surface at Sheep's Tor [2], and at 40 feet below the surface at Burrator, the two sites being quite close together. China clay might occur not only at the surface, but beneath a perfectly hard crust of granite, and it was not safe to assume that the granite would get progressively harder with increased depth. The occurrence of such soft patches of disintegrated granite might be due to water, under pressure from below, containing fluorine and boron arising in past ages through veins.

In view of the change of site at the Gorge dam due to the decomposed granite 50 feet below the surface, mentioned on p. 182, the Authors' views as to why cementation was not tried or adopted would be valuable. Some further particulars of the rock used for the aggregate, and whether or not crushed granite from the site was suitable, would be of interest.

The Authors, in reply, said Sir Clement Hindley had referred to the time taken in the completion of the scheme, namely 6 years. As was pointed out in the Paper, the improvement of the access road, anti-malarial work, and the construction of camp accommodation, offices, etc., were all

[1] G. M. Binnie, "Model-Experiments on Bellmouth and Siphon-Bellmouth Overflow Spillways." Journal Inst. C.E., vol. 10 (1938–39), p. 65. (November 1938.)

[2] E. Sandeman, "The Burrator Works for the Water-Supply of Plymouth," Minutes of Proceedings Inst. C.E., vol. cxlvi (1900–1901, part IV), p. 2.

in progress while the exploratory investigations were in hand, and later, as the type of dam to be adopted became more definitely settled, the large amount of plant required was ordered and installed, so that by the time the detailed plans were received from England everything was ready for active construction.   Sir Clement also raised the question of cost.   The cost of the work was not referred to in the Paper because the dollar fluctuated so much as against sterling, and it would have been possibly more misleading than informative to have given any details.   When Mr. Gourley was in Hong Kong in 1930 the dollar stood at 1s. 3d. and it had varied from 11d. or 1s. up to 3s. or 4s. although not over so wide a range during the time that the scheme was being carried out.   It might, however, be mentioned that the scheme as outlined was estimated to cost 8,000,000 dollars for construction ; notwithstanding extra works which had to be carried out at the low gaps, the addition of 300,000 dollars for the siphons and their outlet works as described in the Paper, and the anti-malarial work costing 122,000 dollars, the scheme was carried out within the original estimate.   At the rate of exchange prevailing at the time the estimate of 8,000,000 dollars was made, construction was to have cost £500,000.   The fact that, notwithstanding the various extras mentioned, the total expenditure was less than the estimate was unquestionably a tribute to the excellent work of the Resident Engineer, Mr. Hull, and his staff.

Sir William Peel had referred to the anti-malarial work.   That involved the drainage of almost 1,000 acres round the camp and residential sites, with the construction of 22 miles of permanent concrete drains.   By making those drains of a permanent type rather than by relying upon cutting earth ditches, maintenance charges were reduced to a minimum, and ultimate economy was thereby achieved.   The expropriated paddy fields which lay within mosquito striking-distance of the camp called for measures more extensive than usually was the case in the tropics, but the 122,000 dollars spent on the work kept the incidence of malaria within reasonable limits, so reasonable that it had not been necessary to pay extra wages to labour as compared with those paid to similar labour in other parts of the Colony.   Quite apart from considerations of health and on merely mercenary grounds, it was economy in a malarial district first to tackle the question of swamp drainage and then to keep a small anti-malarial gang under a foreman trained in that work for maintenance and, where necessary, oiling seeps and small pools which construction work brought in its train.   The neglect of those precautions in one case known to the Authors caused labour rates eventually to advance by nearly 100 per cent. and led to considerable delay in the completion of the work.

Mr. Halcrow suggested that the thrust-block might have been to some extent, if not entirely, dispensed with had the upstream face been made flatter.   It was in some measure a question of economics, for had the upstream face of the rock-fill been made on a flat slope more or less similar

to that adopted on the downstream side, the quantity of rock-fill would have been substantially doubled, and the yardage of the rich concrete of the diaphragm would have been almost doubled, for its thickness at corresponding depths would have remained the same. It was with those considerations in mind that the section as described was adopted as the most economical, having regard to the unit costs of the various items involved. There was the further advantage that by having the thrust-block it was possible to arrange for ready access for inspection, etc., of the diaphragm, and it was doubtful if that would have been practicable had the diaphragm rested directly upon the rock-fill, which was bound to settle somewhat, and certainly irregularly. It might be added that the idea of using a diaphragm in the Gorge dam was suggested by the remedial works on the Norwegian dams to which Mr. Halcrow had referred.

Mr. Halcrow was not happy about the setting of the 20-foot lifts of concrete in the diaphragm, and raised a number of points about the shuttering for the work, which had been almost entirely answered by Mr. Geoffrey Binnie's contribution to the Discussion. The fact that the panels had shown themselves to be absolutely impervious and the joints water-tight proved that the concrete had set properly, and that all the measures involved in the construction of the diaphragm had been successful.

Mr. Halcrow and Mr. Davidson had referred to the " Callendrite " sheeting. Personally, the Authors saw no reason why that sheeting should not have a very long life in the situations in which it had been employed in the dam. In every case it was between masses of concrete, subjected to fairly heavy pressure, not exposed to light and always kept in a reasonably moist condition; indeed, the circumstances were almost ideal for longevity.

Mr. Halcrow and Mr. Temple had expressed the view that cracking of the rock-fill following an earth-tremor or earthquake would allow the escape of sand and lead to loss of support to the thrust-block by the sand-wedge. It should be at once said that the kind of earth-tremor envisaged by the design was not such as more than to vibrate or jar the structures involved, and certainly not to cause any material displacement. The effect of such movement would be somewhat to consolidate the sand without, however, causing it to cohere, although, in view of its coarse nature, and even if the mortar jointing adjacent to the sand were to crack, no appreciable loss of sand or of support was anticipated. If an earth-tremor of greater violence occurred, such as to cause a noticeable drop in the level of the top of the sand-wedge and also of the rock-fill above, it would be readily detected and without undue difficulty or expense that rock-fill could be removed, the sand " topped-up," and the rock-fill reinstated.

Mr. Halcrow and Mr. Wilson mentioned the records of deflexion. There could be no question of cantilever deflexion below the level 453, for below that level the thrust-block and cut-off wall were securely held by the sides of the gorge, and it was probable that the loading and conse-

quent compression due to the depth of water in the reservoir in part
explained the upstream deflexion of that portion of the dam, although it
was to be observed that the maximum deflexion occurred about 4 months
after the reservoir was full. He understood that in the Boulder dam the
conclusion was reached that the load of water on the bed of the valley
had caused general compression. The downstream deflexion of the upper
part of the dam did follow more closely the rise and fall of the reservoir,
although with some lag-effect, and it was of interest to note the decrease
of deflexion as the reservoir fell from September 1937 onwards, indicating
the elasticity of the structure. In reply to Mr. Wilson, no stress–strain
investigation of the rock-fill had been made.

The Authors were much obliged to Mr. Creasy for giving the short
history leading up to the carrying out of the scheme. There was no doubt
that for many years past Hong Kong had to construct dams in valleys
which had poor storage potentialities, and never until the completion of
the Shing Mun scheme had there been a material interval of non-con-
struction ; notwithstanding that, there were many years in which
restrictions had to be imposed.

The Authors thanked Mr. Geoffrey Binnie for his remarks which
supplemented the information given in the Paper.

Although the foundations of the dam were in granite, the overlying
material in the catchment area was laterite, free from vegetation and
giving rise to " flashy " conditions of river-discharge.

Mr. Stuart suggested that the concrete at the base of the thrust-block
would be impaired by percolation if the water were soft. The water was
moderately soft, but, as pointed out in the Paper, the leakage from the
reservoir when full did not exceed 430 gallons per hour. As the water
rose at the greased-paper joint between the thrust-block and the richer
concrete facing, the head measured between the base of the thrust-
block at the deepest part and the drainage gallery could not exceed
100 feet, and as the base of the thrust-block was 80 feet wide, it was not
to be expected even at that section that there would be any material
seepage below the thrust block, the more so as any weak places in the
granite upon which the thrust-block rested were strengthened by grouting.
There was no evidence that any measurable quantity of water in excess
of that gauged in the gallery gauging-sump had passed downstream of the
thrust-block. Mr. Stuart's fears about excessive settlement of the rock-
fill on the hypothesis of percolation under the thrust-block were groundless.

With regard to the provision made for floods, possibly Mr. Stuart did
not realize—and admittedly it was not made clear in the Paper—that
when the Authors stated that the peak rate of run-off was 5 inches per
hour its duration was of a few minutes only, and as stated in the Paper
that became reduced by the lag effect to 3·7 inches per hour. Those
figures were equivalent to the normal maximum flood of the Floods
Committee's Report, upon which Committee, incidentally, both the

Authors served, and it was to be noted that according to that Report, and for conditions in the British Isles, the peak rate of run-off from an area of 3,000 acres was 0·87 inch. It was further stated in the Paper that leaving a freeboard of 3 feet the flood-discharge arrangements allowed of a maximum rate of run-off of 5·25 inches per hour, so that if the increased lag effect of the extra 2 feet in the reservoir were taken into account, a flood exceeding the normal by almost 100 per cent. might be safely dealt with. The Authors, therefore, entirely disagreed with Mr. Stuart's criticism, and the question of an emergency spillway at Pineapple Pass did not therefore arise.

Mr. Temple attempted to make their flesh creep by painting a most gloomy picture of what would happen if an earthquake such as he described occurred at Hong Kong. If it did, neither the Gorge dam nor any other dam would be left standing. The Colony was not, so far as was known, likely to suffer an earthquake, although there was the possibility that there might be earth-tremors of moderate degree, and it was with that eventuality in mind that the design was evolved. Mr. Temple's comments were based upon the hypothesis of a major earthquake, so it was not necessary to follow him in speculation as to the possible effects on the component parts of the Gorge dam. The Authors did not know whether Mr. Temple was really serious in suggesting the merits of a multiple arch dam nearly 300 feet high: personally, they would consider it the least suitable type to employ in an earthquake country.

Mr. Davidson referred to the hardening of copper due to repeated bending backwards and forwards. Any bending to which the copper strips in that case might be exposed would be of an extremely slight character, and embrittlement was not anticipated. The thickness of the rich mixture of concrete was as shown in Fig. 3, Plate 1, portions " 1 " and " 2 "; it was not a facing, as Mr. Davidson appeared to think, but the whole of the concrete in those portions contained 600 lb. of cement per cubic yard.

In reply to Mr. Hunter, there was no doubt that preliminary subsurface investigations by electrical means were in some cases advantageous, but for important structures they had to be checked by borings, and, in certain formations, by trial pits as well. After reviewing the preliminary work which was carried out before the final site was selected at Shing Mun and its cost in the light of Mr. Hunter's remarks, the Authors did not consider that there would have been any material saving in time or money had geophysical methods been adopted.

The cost of a thin arch dam which necessitated high and heavy abutments had been considered, but its cost was 10 to 15 per cent. more than the adopted design, and apart from that it was not thought as suitable as that design for withstanding earth-tremors.

Whilst Pineapple Pass was not suitable for the discharge of flood water, the use of one of the low gaps was considered as an alternative to the siphons, but the cost was rather more than that of the siphons.

Mr. Richards' question as to whether an inverted cone was tried in the model experiments raised a point of interest; that device was not tried, but when a circular horizontal disk was placed at the water-surface over the model it was found that vortex motion was prevented, due, it was surmised, to the exclusion of air, and that suggested that the inverted cone would perform the same function and in a manner more practicable from a construction point of view, but it would cost more than the curtain wall.

In reply to Mr. Walters, cementation would not have rendered the upper sites suitable or economical, but it was adopted not only to ensure watertightness below the cut-off trench at the Gorge site, but also to strengthen the brown granite upon which the thrust-block was founded. The aggregate used was obtained entirely by quarrying sound granite.

Fig: 1.

Future catchwaters

FUTURE CATCHWATERS

Direct catchment-area
3,000 acres

Indirect catchment-area
3,050 acres

PINEAPPLE PASS DAM.

JUBILEE RESERVOIR.

Catchwater

GORGE DAM.

North conduit

Access road

North tunnel

South
conduit

Shek Lai Pui reservoir

South
tunnel

KOWLOON-CANTON RAILWAY.

Catchwater

Reception
reservoir

Kowloon reservoir

Shek Lai Pui R.G. filters 10 M.G./D.

Covered service reservoir

Kowloon S.S. filters

Pipers hill covered service reservoir

Byewash
reservoir

Two 24" trunk mains

Kowloon Tong service reservoir

Stonecutters
Island

Two 24" trunk mains

KOWLOON

Yau Ma Ti service
reservoir

12" and 18" dia.
mains

Scale: 1 inch = 2 miles.

Mile 1   0   1   2 miles.

Mainland resources:—Kowloon water works shown thus:—
Island resources:—Eastern   shown thus:—
Island resources:—Western   shown thus:—
Island resources:—Wongnei Cheong   shown thus:—

VICTORIA HARBOUR

LYEMUN PASS

VICTORIA

HONG KONG

Service reservoirs

Pumping
station

Pokfulum
reservoir

Aberdeen upper reservoir

Tytam
reservoir

Aberdeen lower reservoir

Wong Nei Chong
reservoir

Byewash
reservoir

Tytam Tuk
reservoir

Tytam
intermediate
reservoir

PLAN OF HONG KONG AND DISTRICT.

Fig: 3.

T.W.L. 625.0   635.0   622.0

573.0

Face set in
cement mortar

Ventilating and
plumb-line pipe

Inspection
gallery

453.0

550.0

1 in 1.5

461.0

Squared pitching

380.0

300.0 O.D.

Scale. 1 inch = 160 feet.
Feet 10 0   50   100   150 feet
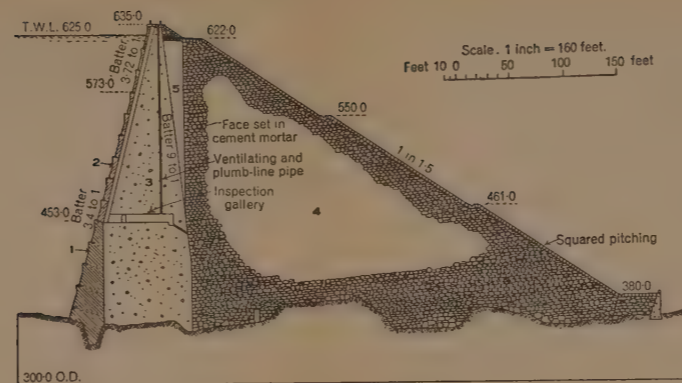
CROSS SECTION OF DAM.

Fig: 4.

Left bank   Right bank

Siphons   Top water-level 625.0   Top of dam 635.0

Inspection
galleries

Reinforced concrete diaphragm

Inspection
galleries

Copper strips

Overflow tunnel ◯

Thrust block foundation line

Bottom of cut-off trench

Copper strips

Diversion tunnel ◯

Scale: 1 inch = 160 feet.
Feet 100   50   0   100   200 feet

300.0 above ordnance datum

ELEVATION OF DAM.

Fig: 7.

T.W.L.   620
625.0
630

660

680

700

Access road

Entrance to
diversion tunnel

Valve-
tower

Bellmouth
overflow

Reinforced concrete diaphragm

Cut-off

Entrance to
inspection
galleries

Existing conduit

Rock fill

Berm

Steps

Slope
1 in 1.5

Gutter

Existing

48" dia.
supply pipe

Stone-toe-level

380.0

48" sluice-valve

Venturi
meter

Recorder
house

Feet 10 0   50   100   150   200 Feet
Scale: 1 inch = 160 feet.

PLAN OF GORGE DAM.

PLATE 1.
THE GORGE DAM.

FIG. 11.

Valve-tower

T.W L. 625·0
Intake Nº 1. 514·5
635·0
DAM
Bellmouth overflow
593·6
Intake Nº 2. 576·0

Scale: 1 inch = ·120 feet.
Feet 100   50   0   100 feet

Intake Nº 3. 537·5

Overflow tunnel

Intake Nº 4. 479·75

48" dia. steel standpipe

Scour 410·11 bellmouth

Diversion and overflow tunnel internal diameter 15'. 0"

Tailbay

Tunnel entrance
360·0
Scour-pipe and supply-pipe

SECTION OF DIVERSION TUNNEL.

FIGS: 12.

Junction of diversion and overflow tunnels

410.11 Scour bellmouth

48" dia. C.I. pipes

Scale: 1 inch = 16 feet.
Feet 10   5   0   10   20 feet

VALVE SHAFT

Grout pipes

Cast-iron bulkhead

Grout pipes

Minimum thickness 12"

15'. 0" dia.

Grout pipes

Standpipe

DIVERSION TUNNEL

Perforated metal screen

CONCRETE STOPPING

Concrete stopping

Training wall

48" flat outlet valve

Supply pipe 48" dia. steel pipes

Supply pipe

Rock fill

30" dia. sluice valve

30" dia. sluice valve

48" dia. scour pipe

Scour pipe 48" dia. steel pipes

Scour pipe

SECTION AA.

360·0 O.D.

SECTION BB.

DETAILS OF INLET TO DIVERSION TUNNEL.

W. J. E. BINNIE and H. J. F. GOURLEY.

ORDINARY MEETING.

7 February, 1939.

WILLIAM JAMES EAMES BINNIE, M.A., President,
in the Chair.

The Scrutineers reported that the following had been duly elected as

Associate Members.

SAMUEL ASQUITH, B.Sc. (Eng.) (*Lond.*), Stud. Inst. C.E.

FRANK BAXTER, Stud. Inst. C.E.

JAMES KELL LAMIE BLACK, B.Sc. (*Belfast*), Stud. Inst. C.E.

CHARLES FREDERICK BLAIR, B.Sc. (Eng.) (*Lond.*), Stud. Inst. C.E.

ATHOL KENNEDY CARIS.

ROBERT WILLIAM CARLEY, B.A., B.A.I. (*Dubl.*)., Stud. Inst. C.E.

IAN STEWART CHISHOLM, B.Sc. (*Edin.*), Stud. Inst. C.E.

WILLIAM JAMES COZENS, B.Sc. (*Edin.*), Stud. Inst. C.E.

IVOR HERBERT DAVIES, B.Sc. (Eng.) (*Lond.*).

ALAN LIONEL JAMES DAVIS.

KENNETH DENING DUDLEY, B.Sc. (Eng.) (*Lond.*), Stud. Inst. C.E.

ERIC HODGKISS FRANKLAND.

ERIK AIDAN OLAF HOLST.

JOHN WHITTAKER HOYLE, B.Sc. (*Manchester*), Stud. Inst. C.E.

ROY CYRIL JENKINS, Stud. Inst. C.E.

JOHN TAYLOR KENDAL, Stud. Inst. C.E.

ELLIOTT NEEDHAM KING, Stud. Inst. C.E.

SIDNEY HOLLIES LEWIS, B.Sc. (*Birmingham*), Stud. Inst. C.E.

WILLIAM DRUMMOND MACKINTOSH, M.C., B.Sc. (*Edin.*).

COLIN ARCHIBALD MacNICOL, B.Sc. (*Durham*), Stud. Inst. C.E.

ROBERT BALLANTINE MUIR, B.Sc. (*Glas.*), Lieut. R.E.

DOUGLAS CAMPBELL MURRAY, Stud. Inst. C.E.

WINSTON FREDERICK JAQUES NEAL, B.Sc. (Eng.) (*Lond.*), Stud. Inst. C.E.

WILLIAM FRANCIS JOSEPH NICHOLSON, Stud. Inst. C.E.

PHILIP HENRY OGDEN, B.Sc.Tech. (*Manchester*), Stud. Inst. C.E.

CHARLES AUGUSTINE HENRY CROSS OGILVIE, B.Sc. (Eng.) (*Lond.*).

FRANCIS HILARY PAVRY, B.Sc. (Eng.) (*Lond.*), Stud. Inst. C.E.

ANTHONY CLAYTON RICE, B.Eng. (*Liverpool*), Stud. Inst. C.E.

PETER RUSSELL, B.Sc. (*Glas.*), Stud. Inst. C.E.

HAROLD KYLE SCOTT, B.Sc. (*Belfast*), Stud. Inst. C.E.

JOHN SMITH, B.Sc. (*Edin.*), Stud. Inst. C.E.

FRANCIS COLIN SQUIRE, B.Sc. (Eng.) (*Lond.*), Stud. Inst. C.E.

DENIS TEMPLE, Stud. Inst. C.E.

ARNOLD THREAPLETON, Stud. Inst. C.E.

ROBERT HENRY RAWLINSON WALKER, B.A. (*Cantab.*), Stud. Inst. C.E.

JOHN WESTACOTT, Stud. Inst. C.E.

JOSEPH WHITTEN.

FREDERICK WILLIAM YOUNG, B.Sc. (*Durham*).

The following Papers were submitted for discussion, and, on the motion of the President, the thanks of The Institution were accorded to the Authors.

Paper No. 5158.

"Some Experiments on the Lateral Oscillation of Railway Vehicles." †

By RALFE DAVIDSON DAVIES, M.A., Ph.D., Assoc. M. Inst. C.E.

TABLE OF CONTENTS

INTRODUCTION.

RAILWAY vehicles, when travelling at high speed, tend to develop a lateral oscillation. When the tires are new this causes little trouble, but as they become worn the oscillation becomes more violent, and finally assumes the characteristic form known as " bogie-hunting." The only cure is to withdraw the vehicle from service and to re-turn the tires to their original profile.

It has long been known that the trouble is connected with the coning of the tires, and the investigation here described started from that point. The investigation is mainly concerned with four-wheeled vehicles and bogies; it does not deal at all with locomotives. It consists of the following parts :—

(1) Preliminary model-experiments on " creep " ; that is, on the departure from pure rolling motion which occurs when there is a horizontal force at the wheel-rail contact insufficient to cause actual slip. As pointed out by Dr. F. W. Carter *, creep plays an important part in the problem because of the acceleration-forces associated with the oscillations.

(2) Model-experiments, about one-fifth full size, with a single axle and with a four-wheeled vehicle. These are described at some length because

† Correspondence on this Paper can be accepted until the 15th June, 1939.—SEC. INST. C.E.

* " Railway Electric Traction," pp. 47 *et seq.*, and pp. 57 *et seq.* London (Arnold), 1922.

they show the characteristics of the motion of coned wheels more clearly than do full-size experiments.

(3) Mathematical analysis. This proceeded at the same time as the model-experiments, checking and being checked by them.

(4) Full-size " crawl " experiments, with a single axle and with a bogie.

(5) Full-size experiments at normal speed on a London Midland & Scottish Railway main line.

These parts will be described in the above order, except that the mathematics, being rather involved, are shown as Appendixes to the Paper. Parts (1) and (2) were carried out in the Engineering Laboratory at Cambridge University, under the supervision of Professor C. E. Inglis, F.R.S., M. Inst. C.E. ; parts (4) and (5) were carried out in collaboration with the L.M.S. Railway, to whom the Author expresses his thanks for permission to publish the results here.

CREEP.

Consider a wheel travelling along a rail. Let the wheel-load be P and let there be a tangential force R at the wheel-tread and in the plane of the wheel. If R/P is zero there is pure rolling, and if R/P is equal to the coefficient of friction the wheel skids or spins. Suppose, however, that the value of R/P lies between zero and the coefficient of friction ; then, due to strain of the surfaces in contact, there is a departure from pure rolling, and a discrepancy between the actual distance travelled and that to be expected from the revolutions of the wheel. Using Carter's definition, this discrepancy will be called the " creep," and the fraction (creep)/(distance travelled) will be termed the " longitudinal creepage." If the horizontal force R at the wheel-tread is perpendicular to the plane of the wheel, then the motion of the wheel in the direction of R, divided by the distance it travels in its own plane, will be called the " lateral creepage."

Fig. 1 (facing p. 226) shows the apparatus for the longitudinal-creep experiments. The rails were of $1\frac{1}{2}$-inch by $\frac{3}{8}$-inch bright steel placed on edge, with the top machined square ; they were held in chairs which could be tilted to vary the rail-cant, without altering the gauge of $11\frac{5}{8}$ inches. The model-truck had cylindrical steel wheels, all ground to the same diameter. Those on the near side were fixed to the axles and all measurements were taken on them ; those on the far side were free, so as to avoid unknown creepage due to non-parallel axles. The back axle carried a brake-drum to which a known couple was applied by two spring balances and a piece of string ; this caused a known force R at the tread of the near back wheel.

Projecting from the inner face of each near-side wheel was a ball-headed pin, which in each revolution passed through a fork fixed to the body of the model. At the start of each run the model was held against a chock, and the wheels were turned until the pins were just below the forks ; a steel ball was then placed in a countersunk hole in each fork, and

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a micrometer measurement was taken over this ball and the head of the pin. This determined the relative angular position of the wheels. The model was then towed along the rails by a string, fixed to it close to rail-level and in the plane of the near-side wheels, that is, in the line of action of the tangential force R. At the end of a measured course the model was stopped, with the pins in suitable positions, and the micrometer measurements were repeated. Since the front wheel had been rolling free while the back wheel was braked, any change in their relative angular position was due to creep. The correction for any difference in circumference was found from an unbraked run. The error in measuring the creep was probably less than 0·002 inch, corresponding to an error in the creepage of 0·00001 on the usual run of 200 inches. The measurement of R by the spring balances was less satisfactory, and some of the smaller readings may contain errors of 10 or 15 per cent.

For the lateral-creep experiments the track was banked to a known angle by packing under the sleepers, the tangent of this angle giving the ratio R/P. All wheels were fixed to their axles and the model was ballasted to load them all equally; the spring-balance gear was removed. Tow-lines fixed to the centre of the model at rail-level were taken through eyes fixed at rail-level to the centre of the track at each end.

At the start of each run, the lateral position of the model was determined by micrometer measurements from the outside of the near rail to the inside of each near wheel. The model was then towed backwards and forwards a certain number of times over a course of known length, and finally stopped at the starting point, where the micrometer measurements were repeated. The mean of the changes in measurement at the front and back wheels was taken as the lateral creep.

In both the longitudinal- and the lateral-creep experiments the wheel-load was varied by ballasting the model, and the wheel-diameter by fitting alternative sets of wheels. Nominal point-contact between wheels and rails was obtained by giving the rails a cant of 1 in 20, so that only their edges touched the wheels, and nominal line-contact by setting their running surfaces as truly horizontal as possible.

The curves in *Fig.* 2 (p. 227) show the relation between R/P and the longitudinal creepage for three different values of the wheel-load, the wheel-diameter being constant. The curves in *Fig.* 3 (p. 228) show the relation between the same quantities for two different wheel-diameters, the wheel-load being nearly constant. The speed was about 2 feet per second throughout. Some of the points are erratic, as is to be expected with the crude apparatus employed, but the shape of the curves is not in doubt. At first they are practically straight, that is, the creepage is proportional to R/P, but at about $R/P = 0.05$ the limit of proportionality is reached and the curves bend upwards, becoming steeper and steeper and finally, it may be supposed, asymptotic to the line $R/P =$ the coefficient of friction (about 0·125 by direct measurement with the wheels

Fig. 1.

MODEL FOR LONGITUDINAL CREEP EXPERIMENTS

Fig. 4.
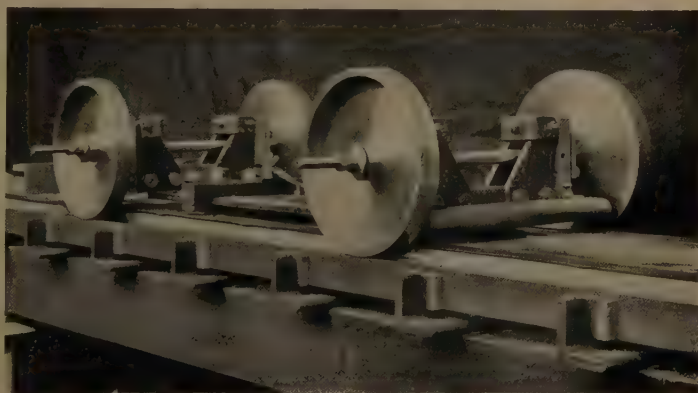
MODEL-VEHICLE ON THE TRACK.

Fig. 5.
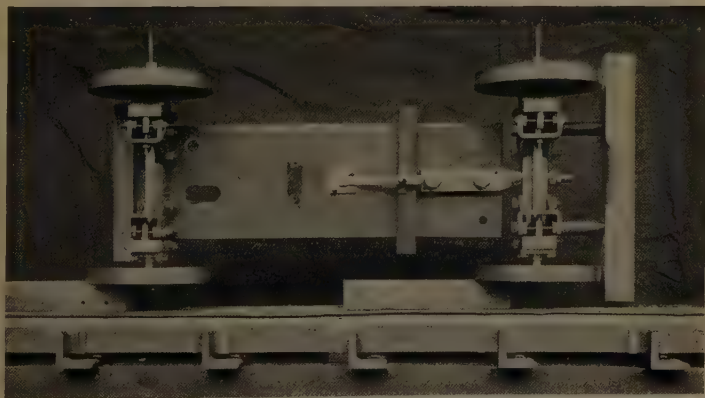
PLAN VIEW OF MODEL-VEHICLE.

Neither longitudinal nor lateral creepage was much affected by changing from nominal point-contact to nominal line-contact between wheels and rails. This is surprising, as it would be expected that the change would be equivalent to reducing the wheel-load; perhaps the rails were not accurately-enough levelled to obtain anything approaching line-contact under the small wheel-loads used.

Fig. 3.
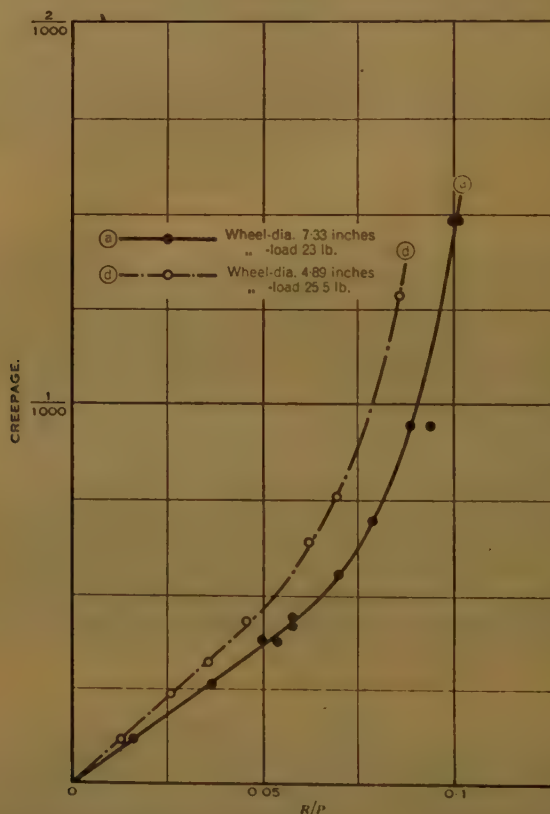
LONGITUDINAL-CREEPAGE CURVES FOR VARIOUS WHEEL-DIAMETERS.

The results of these experiments are used in the calculations given in the Appendixes (pp. 252 *et seq.*). So long as R/P is less than about 0·05 it is assumed that $R = PK$ (creepage), where K is a ratio depending only on the wheel-diameter and load. Its value for the model described in the next section is taken as 155.

No full-size experiments on creep were made. It is to be hoped that the same law holds, but it is not justifiable to extrapolate from the

model-results to find a full-size value of K, as the differences of size, wheel-load and contact-conditions are too great.

Dr. F. W. Carter [1] has calculated a relation between the longitudinal creepage and R. His result is, in the notation of this Paper,

$$K = 3,500 \left(1 + \sqrt{1 - q}\right) \sqrt{\frac{rl}{P}}$$

where the units are inches and lb., l denotes the lateral width of the wheel-rail contact, r denotes the wheel-radius, and q is the ratio of R to its limiting (skidding) value.

This expression agrees fairly well with the experiments as regards the variation of K with r and P. The value of l in the experiments is unknown, but if both the experiments and the expression are correct for the lower values of R/P, l works out to about 0·0035 inch, which is of the right order for the "point-contact" condition. Although the term in brackets indicates a steady upward curvature in the graph connecting R/P and creepage, it does not give the definite bend when R/P is about 0·06, which is a decided feature of all the experimental curves; nor does it make K fall off at the higher values of R/P fast enough to satisfy the experimental results.

MODEL-EXPERIMENTS ON LATERAL OSCILLATION.

(a) *Apparatus.*

The track was straight, 60 feet long and of $11\frac{5}{8}$-inch gauge. The road-bed, of 12-inch by 4-inch steel channel, rested, web uppermost, on strong timber trestles. Angle-iron sleepers, bolted across it, had rectangular slots milled in their vertical legs. Into these slots the rails, of $1\frac{1}{2}$-inch by $\frac{3}{8}$-inch bright steel, were keyed with hardwood wedges; their tops were machined flat. The track was as true as possible short of mounting it on special concrete foundations.

Down the centre of the track was fixed a wooden board, to which was stuck a strip of drawing paper coated with paraffin wax. As the model ran down the track, a spring-loaded stylus made a trace in the wax and thus recorded its path. When this record had been measured it was quickly erased with an electric iron in readiness for the next run. Strips of hoop-iron, screwed to the edges of the board, prevented the wheels of the model wandering so far as to derail, this being necessary because the wheels had no flanges.

The model used for most of the experiments was a four-wheeler, with a

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[1] " On the Action of a Locomotive Driving Wheel," Proc. Roy. Soc. (A.), vol. 112 (1926), p. 151 ; and " On the Stability of Running of Locomotives," Proc. Roy. Soc. (A.), vol. 121 (1928), p. 587.

wheelbase of 23¼ inches, and weighed 60 lb.   The wheels were ground to a coning angle of 1 in 20 and a mean diameter of 7·48 inches. *Fig.* 4 (facing p. 226) shows this model on the track, whilst the constructional details will be most easily understood from the plan view in *Fig. 5* (facing p. 226).   The wheels were fixed to the axles; these turned in self-aligning ball bearings mounted in swing-links universally jointed to the frame, and thus had freedom of translational and angular movement in the horizontal plane.   These movements could all be limited by stop-screws in the four stirrups inside the swing-links.   This arrangement reproduced full-size conditions, where the axles are free within the limits of the axle-box and bearing clearances.   To ensure equal wheel-loads, the rear axle (on the right in *Fig.* 5) was mounted on a bar, which was pivoted longitudinally to the frame of the model.   The wooden saddle, on the extreme right in the Figure, engaged with the catapult mentioned below. The top of the spring-loaded stylus can be seen in the centre of the model.   Special styluses could be fitted to record the paths of individual axles, either as parts of the model, or when tested by themselves.

A rubber-rope catapult, fitted with suitable firing-gear, started the model at any desired speed, and a small slope on the track ensured that this speed was maintained.   Until well clear of the catapult, the model was controlled by adjustable guides, so that it could be started on its run either central on the track or with any desired offset.

After each run, the position of the stylus-line in the wax was read against the offset scale of a special measuring trolley, at regular intervals down the track, and the points so obtained were plotted on squared paper. The curves in *Figs.* 6, 8, 9, and *11* (pp. 231, 233, 236 and 238), showing the path of the model under various conditions, were obtained in this way.

Experiments with this model departed from reality in the following respects :—

(1) Since they dealt with a single vehicle, draw-bar pull and the buffer reactions of adjacent vehicles were absent ; these would tend to steady the motion.

(2) Since the tops of the rails were horizontal, the wheels always made contact with their inner edges ; the analysis assumes the same conditions. With unworn tires, the effect of this approximation is negligible ; it will be considered later.

(3) There were no wheel-flanges. This was not such a serious omission as it appears, since the main object of the experiments was to study the early development of the oscillation, before it had reached such an amplitude as to bring the flanges into play. The action of the flanges will be considered later.

(4) As far as possible, the imperfections of real track were eliminated. Perfect conditions were necessary, in order to study the fundamental characteristics of the motion on the short run available.

(b) *Experiments with a Single Axle.*

Suppose that some disturbance displaces the axle to the right of its central position on the track. Then, owing to the coning, the effective radius of the right-hand wheel is increased and that of the left-hand wheel diminished, and the axle will therefore swing to the left; it will overshoot its central position and will then swing back to the right, and so on again and again. The wheel-coning thus produces a centering action which makes the axle wander from side to side of its central position on the track. It is shown in Appendix I (p. 252) that, at low speed, the resulting path is a sine-curve of wave-length $L = 2\pi\sqrt{(rb/T)}$, where r denotes the wheel-radius, $2b$ the gauge, and T the tangent of the coning angle. For the model this wave-length works out to 10·92 feet. The amplitude depends on the initial disturbance.

Fig. 6.
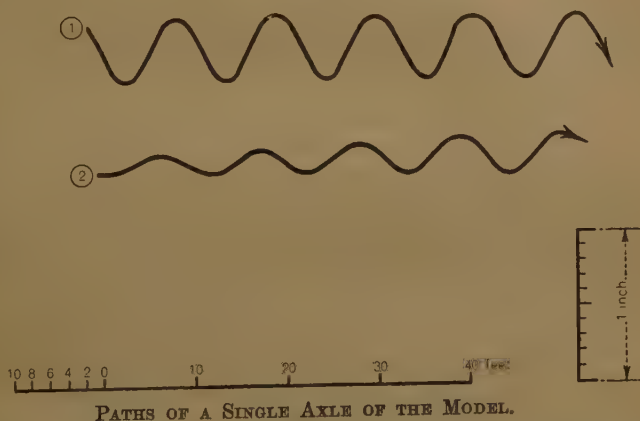
PATHS OF A SINGLE AXLE OF THE MODEL.

Curve 1 in *Fig. 6* was obtained with a single axle, tested by itself, at a speed of under 2 feet per second. The mean measured wave-length is 10·94 feet, and the oscillation neither damps nor builds up.

When travelling at high speed the axle attempts to follow the same path, but to enable it to negotiate the curves, inward accelerating forces must be provided at the wheel-treads. These inward forces are automatically accompanied by outward creep, causing the amplitude of the motion to increase. It is calculated in Appendix I that the amplitude of each half oscillation is $\left(1 + \dfrac{2\pi^2 V^2}{gKL}\right)$ times that of its predecessor; for the model, this "building-up" factor becomes $(1 + 0.00036\,V^2)$, where the speed V is measured in feet per second.

Curve 2 in *Fig. 6* is from a run with a single axle at a speed of 10 feet per second. The wave-length is practically the same as at low speed,

agreeing with calculation. The building-up factor varies between 1·09
and 1·14, averaging 1·085 as compared with the calculated value of 1·036

(c) *Crawl Experiments with a Four-Wheeled Vehicle.*

The mechanics of the motion are studied in Appendixes II and III
(pp. 253 and 256); the results will be summarized here.

If the wheelbase is denoted by $2c$, and the axles have no play in the
frame, the path is a sine-curve whose wave-length is $\sqrt{(1 + c^2/b^2)}$ times
that of a single axle; this will be called the " long " wave, as opposed to
the " short " wave of the single axle.   For the model, $\sqrt{(1 + c^2/b^2)} = \sqrt{5}$,
and the long wave-length is 24·42 feet.

In practice, owing to axlebox and bearing clearances, the axles have
two important plays in the frame : freedom to swing in a horizontal plane
through an angle $\pm \beta$, and end play $\pm \delta$ in the direction of their length ;
δ/c will be denoted by γ. If subject to no other constraint, the axles will
wander in their individual sine-curves as if they were independent, so
long as the amplitudes of these curves do not exceed certain limits fixed
by β and γ.

Actually, within the limits imposed by the clearances, the motions of
the axles relative to the frame are resisted by the axlebox suspensions.
This resistance is probably mostly elastic, any movement of an axle from
its equilibrium position being resisted by forces and couples proportional
to such movement. In *Fig. 24* (Appendix III, p. 256), s is a measure of
these resistances ; as s increases from zero to infinity, the wave-length
changes from short to long, and there is very high damping over an inter-
mediate range.   In the model, the resistances are caused by the inclina-
tion of the swing-links when an axle leaves its equilibrium position, and
are equivalent to about $s = 0·015$, not enough to increase the wave-
length or to cause appreciable damping.

Elastic resistances, however small, equalize the amplitudes of the two
axle motions, and make the front axle lead the back axle in phase, by
about 25 degrees in the model. With this relation between the axle
motions, the clearances allow the axles to move in the short wave, provided
that the amplitude of the motion of the centre of the frame does not exceed
$32·5(\beta + \gamma)$ inches or 94β inches, whichever is the smaller.

Suppose that the model is well outside this " free zone." The axles
will lock hard over inwards and the model will travel in the long wave.
After entering the free zone the axles will release, swinging round in their
clearances through an angle 2β, and the model will leave the free zone at
a reduced angle, and therefore in a long wave of reduced amplitude, as
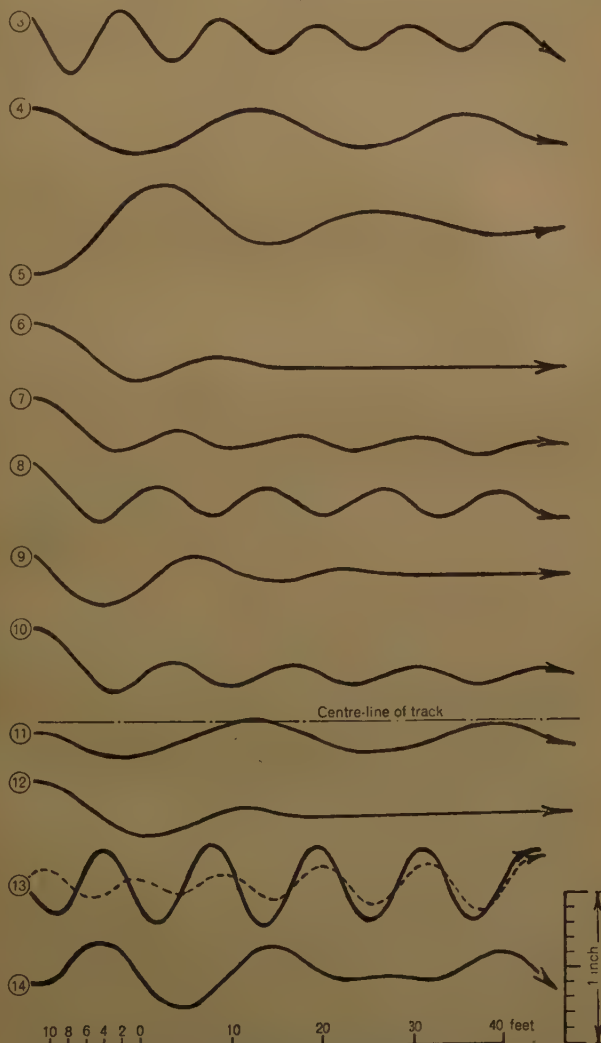shown in *Fig. 7*. Any angular play of the axles therefore damps out the
long wave.

The experiments agreed with the analysis, as is shown by the curves
in *Fig. 8*. These were given by the central stylus of the four-wheeled

Fig. 7.
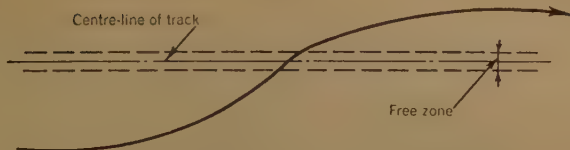
DAMPING EFFECT OF SMALL ANGULAR PLAYS.

Fig. 8.

PATHS OF FOUR-WHEELED MODEL MOVING AT A CRAWL.

model moving at a crawl (2 or 3 feet per second), except in the case of curve 9. In curve 3, β and γ were both large, giving plenty of room for the short wave. The axles probably started in an unnatural phase relationship, and the initial damping marks the quick suppression of a large secondary oscillation. In curve 4, β and γ were both nominally zero and the long wave is obtained; the measured wave-length is about 24·5 feet. Actually, the inevitable small bearing plays produced a small value of β, resulting in the damping action. In curve 5, $\beta = 0.00033$ and $\gamma = 0$: the long wave is heavily damped. In curve 6, $\beta = 0.00066$ and $\gamma = 0$: the long wave is soon killed and the short wave is too small to measure. In curve 7, $\beta = 0.00166$ and $\gamma = 0$: the long wave is killed at once, but the short wave has room to develop; in theory its amplitude cannot exceed $32.5\beta = 0.054$ inch, and by measurement it is about 0·05 inch. In curve 8, $\beta = 0.0028$, and $\gamma = 0$: the short wave has more room; the theoretical maximum for its amplitude is 0·09 inch, which is its actual value.

With small end plays as well as small angular plays, the axles seemed to take up the angular plays first, after which they could only use the end plays by sliding the axleboxes across the guides. This they might or might not do; in consequence the curves, although similar to those for angular play only, are not so consistent. The damping of the long wave is as pronounced as ever, but the wave-length and amplitude of the short wave are rather uncertain, and it is usually slightly damped.

In curve 9, $\beta = 0.0004$ and $\gamma = 0.0004$: theoretically, this limits the amplitude of the short wave to 0·026 inch; actually it is invisible. The speed on this run was 10 feet per second, which makes the high damping all the more meritorious. In curve 10, $\beta = 0.00166$ and $\gamma = 0.0017$, limiting the amplitude of the short wave to 0·011 inch; actually it starts at 0·09 inch, and is slightly damped.

The ideal is that, after any chance disturbance, the vehicle should return to the centre of the track and should stay there. Very small angular plays, with or without very small end plays, seem to achieve this at low speed.

In curve 11, β and γ were both zero, but the axles were out of parallel by an angle $2\alpha = 0.0017$ radian, as accurately as could be measured with micrometers. According to Appendix III (B) (p. 257), the axis of motion should be displaced from the centre-line of the track by $\dfrac{cr\alpha}{bT} = 0.127$ inch; the actual displacement is 0·11 inch.

According to Appendix III (C) (p. 258), if the axles have end play, suitably resisted, and no angular play, the oscillation will be heavily damped. The arrangement was tried, using springs of various strengths to control the end plays; curve 12 shows about the heaviest damping obtained.

Curve 13 shows the relation between the axle motions, with full angular

and no end plays; the full line refers to the front axle and the dotted line to the back axle. At the start the axles were set unnaturally, but they soon adjusted themselves to approximate equality of amplitude, with the front axle leading in phase by 20 or 25 degrees. The longitudinal co-ordinates refer to the position of the centre of the model, not to the positions of the individual axles.

It has so far been assumed that the control exercised by the suspensions is elastic. If it is assumed to be frictional, and calculations are made as in Appendix III (pp. 256 *et seq.*), an equation of motion is obtained which represents two undamped waves of different frequencies. In curve 14, full angular but no end play was given, and the pivots of the suspension-links were tightened to introduce friction. The run is too short to be conclusive, but it suggests a beat.

(d) Experiments with a Four-Wheeled Vehicle at Speed.

Speed makes the oscillation increase in amplitude, as with a single axle. In Appendix IV (pp. 260 *et seq.*) it is calculated that, if the axles have full angular but no end play, the amplitude of each half oscillation is $\left\{1 + \dfrac{\pi^2 V^2 (1 + p)}{gKL}\right\}$ times that of its predecessor, where p is a constant (less than unity) depending on the dimensions of the vehicle; for the model, this building-up factor becomes $(1 + 0.00024V^2)$, where the speed V is measured in feet per second.

In curve 15 of *Fig. 9* (p. 236), the axles had full angular but no end play; the speed was 12 feet per second: the model moves in the short wave and the oscillation builds up, rather irregularly. In curve 16, the axles had full angular and full end play; the speed was 10 feet per second: again the model moves in the short wave; the average value of the building-up factor is 1·13, as compared with the calculated value of 1·024. In curve 17, the axles had nominally no plays; the speed was 20 feet per second: the long wave is obtained, as in *Fig. 8*, curve 4 (for the same conditions at a crawl), but the damping effect of the small plays actually present in the bearings is now overcome by the speed, and the motion builds up.

The axles were then given small angular plays of $\beta = 0.00066$, but no end play. In curve 18, at a speed 13 feet per second, the model is still under control. In curve 19, at 20 feet per second, the model is getting out of control; this curve should be compared with curve 6 (*Fig. 8*), for the same conditions at a crawl.

The addition of end play did not help. In curve 20, $\beta = 0.00066$ and $\gamma = 0.00066$; the speed was 20 feet per second. The device of giving end play only, resisted by springs, which was so successful at a crawl, also failed at high speed. In curves 21 and 22, the speeds were 15 and 20 feet per second respectively; the model was adjusted as in curve 12.

Many more runs were made than are referred to here, but no means were found of stabilizing the model at speeds over about 20 feet per

second. This is not surprising: the calculated building-up factor is c[

the form $\left(1 + \dfrac{CV^2}{LK}\right)$, where C depends on the dimensions of the model

above a certain speed this factor is bound to win, since as V increases $\dfrac{CV^2}{LK}$

Fig. 9.
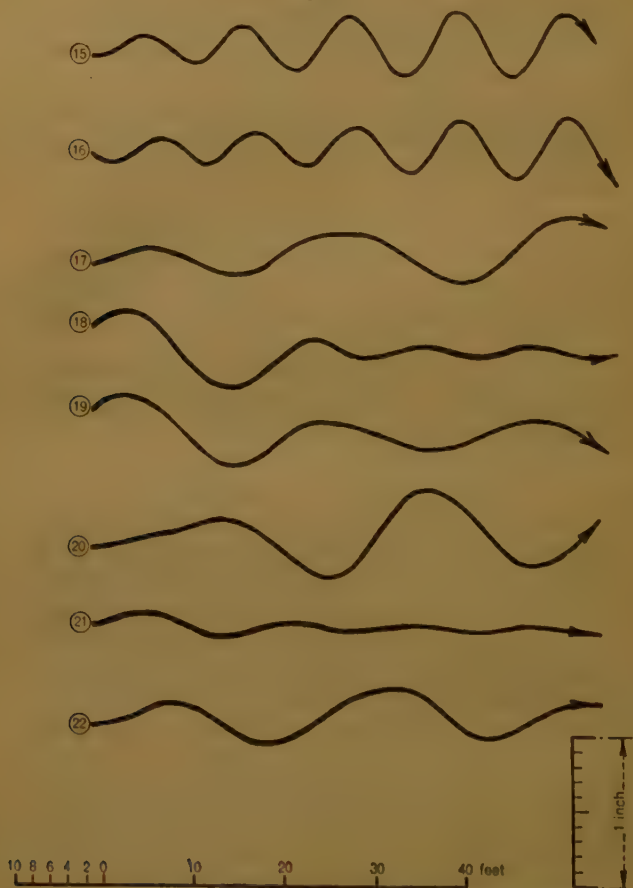
PATHS OF FOUR-WHEELED MODEL MOVING AT SPEED.

becomes overwhelming. To make matters worse, when the speed exceeds
a certain limit, K ceases to be constant and falls off sharply, as explained in
Appendix I (pp. 252 *et seq.*). It will be noticed that the shorter the wave-
length, the greater the building-up factor.

Both with the single axle and with the four-wheeled model, the building-
up factor found by experiment is decidedly greater than the calculated

value. Perhaps the effective value of K falls off at quite low speeds; since the model is unsprung, small vertical irregularities of the rail surfaces would reduce adhesion.

(e) Experiments with a Four-Wheeled Bogie.

Such a bogie is essentially a four-wheeled vehicle, from the centre of which one end of a coach is hung by swing-links and springs. At speed, this suspended load produces dynamic effects.

The shortness of the track made it necessary to idealize the coach as two concentrated masses, one over each bogie, and to use a model representing one such mass and its bogie. *Fig. 10* (facing p. 227) shows this model with the rear axle removed. The lead mass was carried at each end on ball bearings, which ran in grooved rails fixed across the bogie, and was centered by a cantilever spring. The length of the cantilever, and therefore the periodic time of oscillation of the mass, could be varied by moving the spring anchorage on the bogie. An oil dash-pot, like a hydraulic shock-absorber, provided adjustable damping. A recording stylus was fixed to the centre of the mass. The bogie weighed 60 lb. and the mass 85 lb.

For a given setting of the control spring, let the natural period of oscillation of the mass relative to the bogie be t seconds. Let the model be travelling at a speed of V feet per second in a sine-curve of wave-length L feet; then the time of oscillation of the bogie will be L/V seconds. A simple calculation in forced harmonic motion shows that, if there is little damping, $V = L/t$ is a synchronous speed at which the mass oscillates very violently; below this speed the mass moves in phase with, and more than, the bogie; above this speed, it moves in antiphase, with an amplitude which becomes smaller and smaller as the speed rises; at twice the synchronous speed, the movement of the mass is less than half that of the bogie. The addition of damping modifies the phase-relations and keeps the mass under control at the critical speed, but it increases the movement of the mass at speeds well above the critical.

Below the critical speed the oscillation of the bogie should build up rather faster than that of a four-wheeled vehicle, since the mass is moving in phase with, and more than, the bogie; above the critical speed the antiphased oscillation of the mass should tend to suppress that of the bogie.

In all but the last two curves of *Fig. 11* (p. 238), the axles of the model had full angular but no end play, so the bogie moved in the short wave; the addition of end play was found to make little difference.

In curves 23 and 24, the natural period of oscillation of the mass was 1·37 second, corresponding to a critical speed of 8 feet per second; there was no damping, except for the slight friction in the ball bearings. The full line shows the path of the mass and the dotted line that of the bogie. In curve 23, the speed was 8 feet per second, the critical speed; the oscillation of the mass builds up rapidly. In curve 24, at a speed of 19 feet

per second, the mass is much steadier; the oscillation at its natural frequency, which predominates in the record, would die away on a longer run.

Fig. 11.

PATHS OF MODEL-BOGIE.
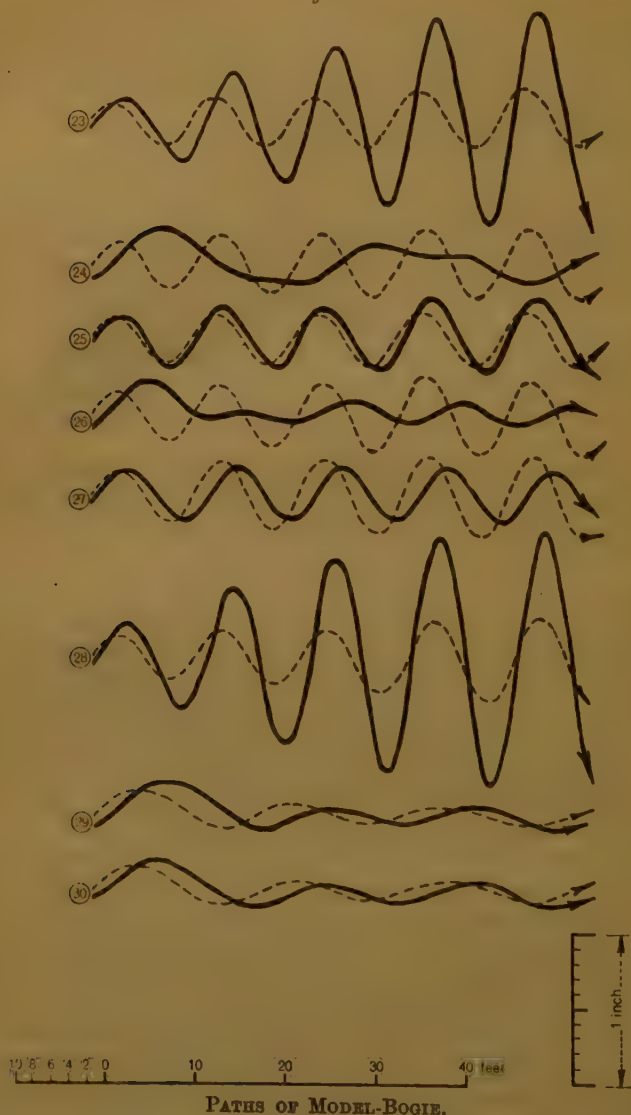
Curves 25 and 26 are repetitions of 23 and 24, with moderate damping added; there is a great improvement at the critical speed. Increasing the damping to dead-beat makes little further improvement at the critical

speed, and is harmful at higher speeds; it comes too near to locking the mass to the bogie, a reversion to the four-wheeled vehicle and all its vices. Curve 27 shows this, the conditions being a repetition of those for curve 26, but with dead-beat damping.

In curve 28, the natural period of oscillation of the mass was adjusted to raise the critical speed to 13·7 feet per second; there was no damping. The speed of the model was 12 feet per second, well below the critical speed, but the oscillation was so dangerously violent that a run at the critical speed was not attempted. This shows that the higher the critical speed, the more violent the oscillation it produces, which is to be expected.

Since, above the critical speed, the oscillation of the bogie does not build up as fast as that of a four-wheeled vehicle, it is more easily controlled by the devices which were effective at a crawl. This is shown by curves 29 and 30. In both cases the actual speed was 19 feet per second, and the critical speed for the short wave 8 feet per second; the mass was moderately damped. In curve 29, $\beta = 0.0004$ and $\gamma = 0.0004$. In curve 30 the axles had full end play, resisted by springs, and negligible angular play, as in curves 12, 21, and 22.

COMPARISON OF THE MODEL-EXPERIMENTS WITH REALITY.

a) Speed-Scale.

Comparing the model with a full-size vehicle, for equivalent speeds the paths must be similar; that is, the building-up factor, and therefore $\dfrac{V^2}{LK}$, must be the same. The speed-scale is therefore the ratio of the values of $\sqrt{LK}$ in the two cases, and cannot be determined, since the full-size value of K is not known.

b) Wheel-Rail Contact Conditions.

The wheels of the model were true cones, and they always made contact with the inner edges of the rails; the analysis in the Appendixes assumes the same conditions.

In reality, with unworn tires and rails, the angle of cant and the coning angle are both 1 in 20, the rails are at 59·25-inch centres, their running surfaces have a 12-inch radius, and the flanges limit the transverse displacement of an axle to $\pm \frac{3}{8}$ inch. The maximum tilt of an axle is therefore $\dfrac{2 \times 0.375}{20 \times 59.25}$, and the maximum transverse displacement of the wheel-rail contacts is $\mp \dfrac{2 \times 0.375 \times 12}{20 \times 59.25} = \mp 0.0076$ inch full size, which is negligible. The model and the analysis are therefore, in this respect, a fair representation of reality for unworn tires and rails.

Fig. 12 (p. 240) shows diagrammatically the profile of a worn rail. Rails

of dates ranging from 1914 to 1934 were examined on straight stretches of
L.M.S. fast main-line track. The radius of the part AB of the profile
varied from 12 inches in some cases to $13\frac{3}{4}$ inches in others. To the left
of A (distant about $\frac{3}{4}$ inch from the inner edge), the radius decreased
progressively. Evidently, throughout the life of a rail, the profile of the

Fig. 12.

DIAGRAMMATIC PROFILE OF WORN RAIL.

central part of the running surface hardly changes, the inner edge is slightly
modified, and the radius at the outer edge disappears as the rail wears
down.

Fig. 13 shows diagrammatically the profile of a worn tire; in a bad
case the radius of the part CD may be as small as 12 inches. The unworn
profile is shown dotted.

Evidently, as would be expected, tires and rails wear to fit one another.
Suppose, for simplicity, that both wear to circular cross sections, the

Fig. 13.

DIAGRAMMATIC PROFILE OF WORN TIRE.

radius of the rail section (convex) being R_1, and that of the wheel-section
(concave) R_2, slightly greater than R_1. Consider a single axle, as on
pp. 231 et seq. and in Appendix I (pp. 252 et seq.). If the axle is dis-
placed $+ y$ laterally from its central position on the track, it tilts through
an angle of approximately Ty/b, where T, the tangent of the coning
angle, is measured at the centre of the tire-tread. The wheel-rail contacts
therefore move $- RTy/b$ laterally, where $\dfrac{1}{R} = \dfrac{1}{R_1} - \dfrac{1}{R_2}$, making the effec-
tive radii of the two wheels differ by approximately $2T\left(1 + \dfrac{RT}{b}\right)y$,
instead of by $2Ty$ with unworn tires.

Thus, although wear does not usually much alter the actual mean value of T, it increases its effective value $\left(1 + \dfrac{RT}{b}\right)$ times, and therefore reduces the wave-length of the oscillation to $\dfrac{1}{\sqrt{(1 + RT/b)}}$ of its former value. If R_1 and R_2 are nearly equal, R is very large, and the wave-length is much reduced; the building-up factor is correspondingly increased, since the wave-length appears in the denominator of that factor.

Tire-wear therefore reduces the wave-length and increases the rate at which the oscillation builds up at a given speed.

c) Axle-Plays and Control of Axles by the Suspensions.

The Paper is mainly concerned with the bogies of passenger coaches. Typical dimensions are: wheel-radius, $r = 21\frac{1}{2}$ inches; gauge between rail-centres, $2b = 59\frac{1}{4}$ inches; tangent of coning angle, $T = \frac{1}{20}$, making the short wave-length $L = 59.3$ feet; play of axleboxes parallel to the rails, $\pm \frac{1}{8}$ inch; axlebox guides, centre to centre, about 76 inches, making the angular play of the axles $\pm \beta = \pm 0.0033$; play of axleboxes parallel to the sleepers, $\pm \frac{1}{8}$ inch; end play of journals in their brasses, $\pm \frac{1}{16}$ inch, making the total end play of the axles $\pm \delta = \pm \frac{3}{16}$ inch; wheelbase, $2c = 108$ inches, making $\gamma = \dfrac{\delta}{c} = \pm 0.0035$.

By conditions (19) of Appendix II (p. 255), it follows that the bogie can move in the short wave, provided that the amplitude of the oscillations of the axles does not exceed about 1 inch, which is greater than the limit of $\frac{3}{8}$ inch imposed by the wheel-flanges.

The design of the bogie suggests that, within the limits of the clearances, the control of the axles by the suspensions must be small and mainly elastic, being mostly due to the inclination of the suspension-links from the ends of the springs. The model, with full axle-plays, should therefore be a fair representation of the bogie.

The oscillation of the model was very heavily damped when β and γ were both reduced to about 0.0004. To reach these values in the real bogie would mean eliminating the end play of the journals in their brasses and reducing axlebox clearances to $\pm \frac{1}{64}$ inch, or $\frac{1}{32}$ inch total clearance. This would raise practical difficulties; it might be necessary to lubricate the axlebox guides, and there might be trouble in ensuring that the axles were parallel within such close limits.

The other damping device used on the model, of giving only end play resisted by springs, was tried in the full-size experiments described on pp. 246 et seq.

d) The Action of the Flanges.

Experiments on the effect of flange-impact on locomotives have been made in America, using a four-wheeled model with no axlebox or bearing

16

clearances [1]. The results showed that, above a certain critical speed, the impacts alone were sufficient to maintain an oscillation. The tires were not coned.

In ordinary vehicles and bogies, the clearances given in practice are so large that it would be necessary to begin by studying the behaviour of a single axle.

At a crawl, any action by the flanges must tend to suppress the oscillation. If the left wheel-flange comes into operation, as shown in *Fig. 14*, the rail will exert a forward force upon it, and the axle will therefore turn clockwise; but as soon as wheel and rail are parallel, the contact between flange and rail will cease, and the wheels and axle will start off again on their sine-curve, leaving the rail tangentially. Thus, whatever the angle of incidence, the angle of reflexion is zero.

Fig. 14.
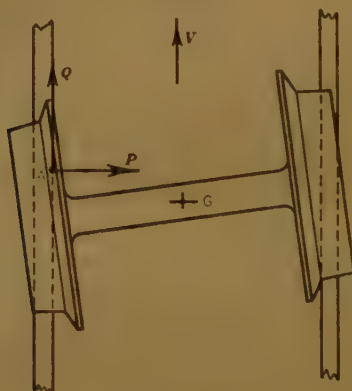
ACTION OF A FLANGE.

An exact analysis for high speed is impossible. Since the rails are not rigid laterally, the impulse on the flange is not instantaneous; also, the position of contact between flange and rail depends on the exact shape of both, and is quite uncertain after a little wear. What follows is therefore only an outline.

In *Fig. 14*, the flange strikes the rail at A, a little in advance of the wheel-centre and a little below the rail-surface. There will be normal and frictional impulses; these may be resolved into lateral and longitudinal impulses, P and Q, and a vertical impulse with which this analysis is not concerned. Taking probable values for the various quantities concerned, it appears that P and Q have a clockwise impulsive moment about G which is roughly proportional to P, and are not otherwise greatly affected by ψ.

[1] B. F. Langer and J. P. Shamberger, " Lateral Oscillations of Rail Vehicles." Trans. Am. Soc. Mech. E., vol. 57 (1935), p. 481.

be angle of incidence. Now P itself must increase with $V\psi$, the lateral velocity of incidence. The axle therefore acquires an angular velocity roughly proportional to $V\psi$, and skids round clockwise. The angle through which it skids cannot be readily calculated, because it is sliding transversely at the same time, but the angle may be expected to increase with V, until finally the angle of reflexion is greater than the angle of incidence, and the action of the flanges helps to build up the motion.

FULL-SIZE CRAWL EXPERIMENTS.

Various single axles and a four-wheeled bogie were used. A straight track, 200 yards long, was laid in the Carriage and Wagon Works at Derby. The first 100 yards consisted entirely of new material, and the second 100 yards of old rails, chairs, sleepers, fishplates, etc., reassembled in the relative positions which they had occupied in service. The whole was given a uniform fall of 1 in 300, to ensure free running.

The recording gear was designed by Mr. Eling Smith, of the Chief Mechanical Engineer's Department of the L.M.S. Railway. *Fig. 15* (facing p. 227) shows the arrangements for a single axle. A tubular frame, pivoted on the ends of the axle, carried a small spring-loaded recording wheel. This wheel ran on a board fixed outside the near rail, and marked on patches of paint applied to that board at intervals. The distances of the marks from the inner edge of the rail were measured with an offset scale. In the experiments with a bogie, the recording wheel was carried on a post fixed on the rear axlebox guide.

Fig. 16 (p. 244) shows the records taken on the new track; they are very different from the clear sine-curves of the model. The track was reasonably straight and true, but even the best-laid track has kinks and waves comparable in magnitude with the possible lateral motion of an axle ($\pm \frac{3}{8}$ inch). The path of an axle depends on the interaction of its natural wave-motion with these track imperfections, and is completely distorted when a flange comes into action. The records are further complicated by being measured from a crooked rail as datum; this error could have been avoided only with considerable trouble and expense.

The records of runs with the same axle or bogie are very similar, differences in starting conditions being quickly absorbed (as shown by comparing records 1 and 2, or 3 and 4). This is almost certainly due to flange-action (as explained on pp. 241 *et seq.*) operating on a crooked rail; after the first one or two traverses, the flanges always lose contact with the rails at the same kinks. The paths of different axles are generally very different, but certain prominent features, corresponding to especially violent kinks in the track, recur.

The paths of the worn wheels confirm the conclusion reached on pp. 239 *et seq.* that tire-wear reduces the wave-length. The first set (curves 5 and 6) show a pronounced oscillation with a wave-length of about 20 feet,

as compared with the theoretical value of 59 feet for unworn wheels of the
same diameter.   The second set (curves 7 and 8) show a similar but less
definite result.

In theory, if the natural motion of an axle or bogie on straight track

Fig. 16.
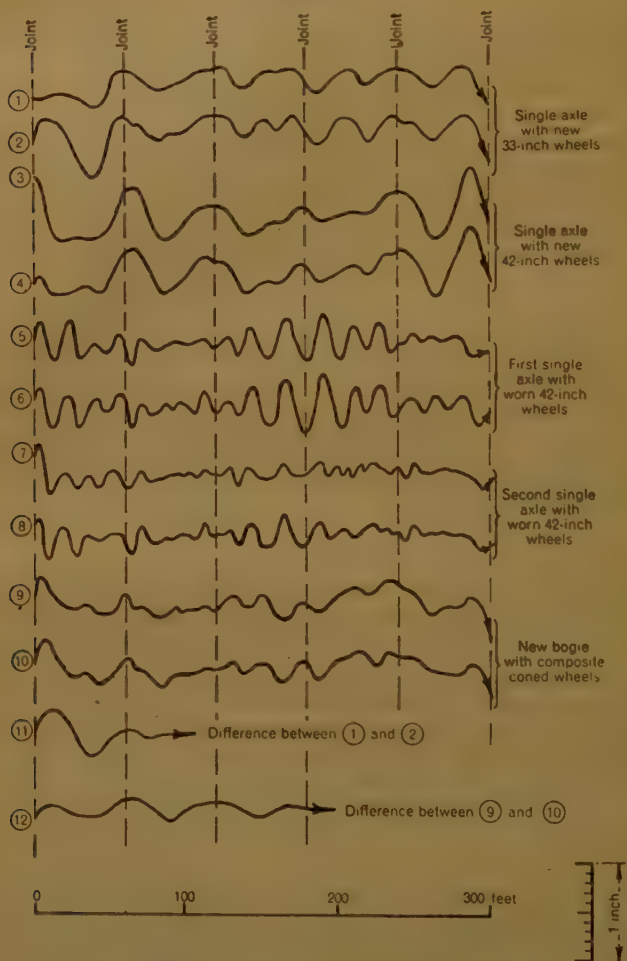
RECORDS OF FULL-SIZE CRAWL EXPERIMENTS ON NEW TRACK.

is a sine-curve, then the difference between two records taken over the
same crooked track gives the natural motion.   Curve 11 is the difference
between curves 1 and 2 : the wave-length agrees well with its theoretical
value of 52 feet; the damping must be caused by the action of the flanges,
as explained above.   Curve 12 is the difference between curves 9 and 10 :

he wave-length agrees well with the theoretical value for the individual axles of the bogie (namely, 59 feet); this confirms that a four-wheeled bogie moves in the short wave.

Fig. 17 shows the records taken on the old track. Here the imperfections of the track itself mask everything else. There is, however, one

Fig. 17.
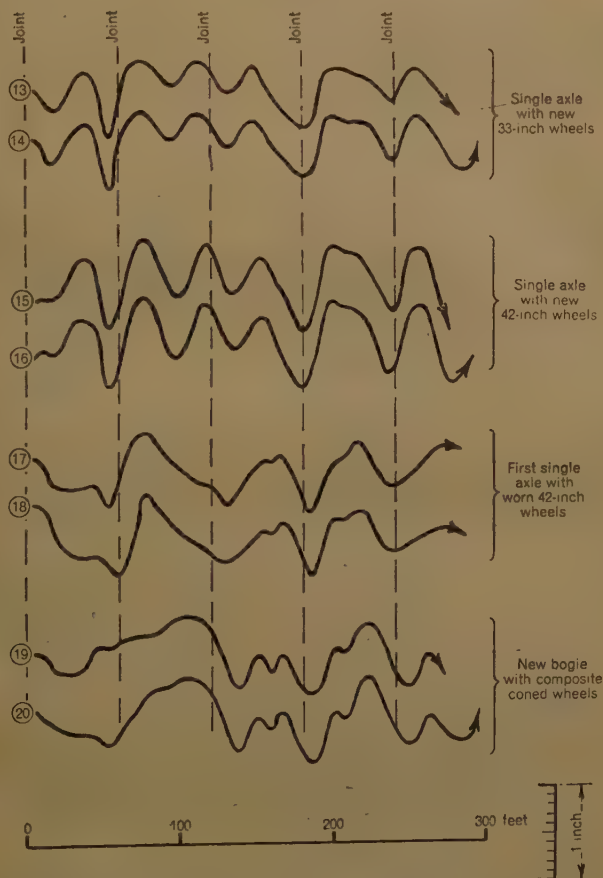
RECORDS OF FULL-SIZE CRAWL EXPERIMENTS ON OLD TRACK.

important piece of negative evidence; the worn wheels did not oscillate at their 20-foot wave-length. It is the fit between wheels and rails, not wear in itself, which reduces the wave-length, and it was noticed that on this track the rails happened to be worn and canted so that they did not fit the actual worn wheels tested. This probably explains why " hunting " is intermittent in practice.

FULL-SIZE EXPERIMENTS AT NORMAL SPEED.

The same vehicle, a third-class corridor brake with four-wheeled bogies, was used throughout. For the main experiments, it was coupled at the rear of a passenger train from Derby to St. Pancras, and the behaviour of its rear bogie, the last of the whole train, was investigated at various speeds. Three different sets of wheels were tried under this bogie, one with new tires and two with worn tires. Accelerometers recording on celluloid, made by the Cambridge Instrument Company, were used; they were fixed on the floor of the coach above the bogie-centre, in the bogie itself close to its centre, and sometimes on the head- and tailstocks of the bogie.

The method had the following limitations. Firstly, since for a given path the acceleration varies as the square of the speed, it was difficult to get recognizable readings at low speeds; secondly, the instruments on the bogie had to have such stiff springs that they were not nearly dead-beat; and thirdly, because of the play between the axles and the bogie-frame, the records did not give the actual motion of the wheels.

Taking the wheels with new tires first, curve 1 in *Fig. 18* is a record from the instrument near the centre of the bogie. The speed was 106 feet per second, the longitudinal scale is 1 inch = 1·0 second or 106 feet, and the lateral scale 1 inch = $4g$. The fundamental wave (dotted) is that of the wheels on the track; it has a wave-length of about 60 feet. The frequency of 5·5 per second, occurring in one or two places, is the natural frequency of lateral oscillation of the bogie-frame on its side bearing springs, etc.; this was verified with the vehicle stationary. The high frequency (imperfectly reproduced in tracing) is the natural frequency of the instrument. Curve 2 is from the instrument in the coach, and was taken almost simultaneously with curve 1. The scales are 1 inch = 1·03 second or 110 feet, and 1 inch = 1·7g. The bogie-wave (60 feet, or 0·56 second long) is superimposed on a longer wave, of length 1·25 second. The latter represents the natural period of lateral oscillation of the end of the coach on its swing-links, etc.; this, too, was verified with the vehicle stationary. The high-frequency ripple must be a vibration in the coach, exaggerated because of its closeness to the natural frequency of the instrument.

With new tires, the bogie-wave did not generally appear at speeds below about 60 miles per hour, and it was never violent. This suggests some damping influence, probably the stiffness of the axlebox suspensions. The wave-length varied from 49 to 63 feet in different records, which agrees with theory (59 feet) as well as can be expected, bearing in mind the effect of track-irregularities; it was not consistently affected by the speed.

Turning now to worn tires, curve 3 is from the instrument at the bogie-centre, during a period of hunting, with the first set of worn tires. The speed was 103 feet per second, and the scales are 1 inch = 0·96 second or 99

Fig. 18.
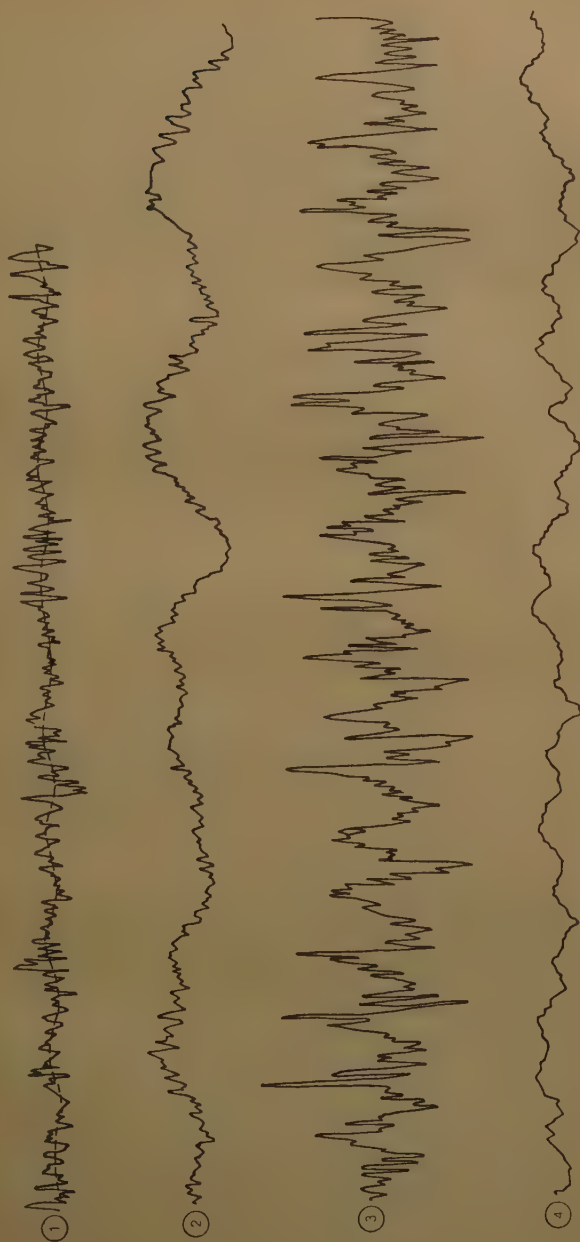
ACCELEROMETER RECORDS TAKEN AT SPEED IN SERVICE-CONDITIONS.

feet, and 1 inch = 4g. The fundamental wave has a much greater ampli
tude than in curve 1 ; its length is 32 feet. Curve 4, taken almost simul-
taneously with curve 3, is from the instrument in the coach. The scales
are 1 inch = 0·93 second or 96 feet, and 1 inch = 1·7g. The bogie-wave
of curve 3, much reduced in amplitude by the coach suspension, is super-
imposed on the natural oscillation of the coach.

With worn tires, the bogie-wave sometimes appeared at speeds as
low as 40 miles per hour, and at higher speeds it often became very violent.
Sometimes, however, there was no sign of it at quite high speeds, probably
because the wheels did not happen to fit the rails. Wave-lengths, from
different records, are given in Tables I and II.

<div align="center">TABLE I.—FIRST SET OF WORN TIRES.</div>

Speed : feet per second . . .	59	73	100	100	101	103	109
Wave-length : second . . .	0·33	0·32	0·32	0·35	0·33	0·32	0·33
Wave-length : feet	20	24	32	35	34	33	36

<div align="center">TABLE II.—SECOND SET OF WORN TIRES.</div>

Speed : feet per second	59	88	103	109	114	116
Wave-length : second	0·32	0·26	0·25	0·26	0·24	0·255
Wave-length : feet.	19	23	26	28	27	30

The constancy of the time wave-length in Table I suggests a resonance
effect. This cannot be the case, however, since, as between the two
Tables, only the tire-profiles differ, yet the time wave-lengths are quite
different ; further, the time wave-length of neither Table agrees with any
natural frequency which could be discovered in either the coach or the
bogie. The explanation must be that the wave-length increases with the
speed so rapidly that the time-frequency remains nearly constant ; some
increase is foretold by the equations in the Appendixes and by the model-
experiments, but once hunting has developed the motion must be largely
governed by the flanges.

In the first set of worn tires the radius of the part CD of the profile
(*Fig. 13*, p. 240) varied from 14 inches to 18 inches, and in the second
set from 12 inches to 13 inches. The second set hunted more violently
than the first, and, as the Tables show, with a shorter wave-length.

By an arrangement of " Bowden " wires and parts of a Hallade instru-
ment, Mr. Eling Smith recorded the horizontal movements of the axle-
boxes in their guides. There was too much inertia in its moving parts
for the instrument to be entirely satisfactory, but it confirmed the wave-
lengths given by the accelerometers. It also showed that the axle motions
were often more than 90 degrees out of phase, which agreed with the fact

that more violent accelerations were recorded at the ends than at the centre of the bogie-frame. Both the phase-relationship and the relative amplitudes of the axle motions were very uncertain, however, and they seemed to be constantly disturbed, probably by track-irregularities and by the action of the flanges.

With the second set of worn tires, a run was made with all the standard axlebox clearances doubled. There was no decided change in the violence of hunting, but the wave-length was slightly reduced. The standard clearances are ample to allow unworn wheels to perform their individual oscillations, but they may interfere slightly when the wave-length is much reduced by wear; also, the axles may not naturally centre in the available clearances. The records of the "Bowden"-wire instrument suggest some interference.

An attempt was made to damp out the bogie-wave, by giving the axles no angular play and resisting their end play, a device which had been suggested by calculation and had proved effective with the model. Four special axleboxes were made. Distance-pieces, each consisting of two 3-inch by 1¾-inch channels riveted back to back, were bolted on to the faces of the two axleboxes on each side, as shown in *Fig. 19* (facing p. 227). The journals had no end play in the brasses, which were rigidly held in the axleboxes. The axles had therefore to remain parallel, and could only traverse across each other by bending the distance-pieces. The standard axlebox clearances were given.

With the second set of worn tires, this arrangement raised the speed at which hunting usually started from about 40 to about 50 miles per hour. On the other hand, once hunting did start, rather more movement reached the body of the coach, perhaps because, the axles being always parallel, their oscillations were nearly in phase. On the whole, there was no real improvement.

CONCLUSIONS.

A single axle, with a pair of coned wheels, wanders along the track in a sine-curve whose wave-length depends on the gauge, the wheel-diameter, and the coning angle. For the sizes of wheels commonly used on vehicles and bogies, this wave-length is between 50 and 60 feet. At speed the lateral motion builds up, and the amplitude of each oscillation exceeds that of its predecessor by a fraction which varies directly as the square of the speed and inversely as the wave-length of the motion. With use, the tires wear hollow, until they nearly fit the surfaces of the rails; this reduces the wave-length of the motion and therefore increases the rate at which the amplitude builds up at speed.

In vehicles and bogies the various clearances given in practice allow the axles to perform their individual sine-curves; the wave-length of the motion of a vehicle or bogie is therefore that of the individual axles. At

low speed there is some damping from the axlebox suspensions, but at
high speed with worn tires this is overcome, and the oscillation becomes
violent.

The Author has not properly investigated the action of the flanges;
he hopes to do this some day with a model. In the full-size main-line
experiments, with new tires the wave-length agreed well with the calcula-
tions based on the coning only. When the wave-length was shortened
by tire-wear and hunting began, the flanges must have had an important
effect on the motion.

Since coning, modified by wear, is the root of the trouble, the obvious
cure is to use cylindrical tires; all centering action would then be lost,
and the wheels would wander at random, checked only by their flanges.
Coning has, however, two advantages which would also be lost: it auto-

Fig. 20.
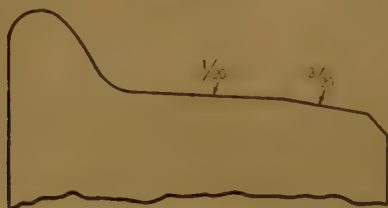
PROFILE OF COMPOSITE CONING.

matically takes up small differences in wheel-diameter by displacing the
centre-line of the motion, and it deals with gentle curves in the same way;
in both cases cylindrical wheels would be thrown against one flange and
creep or slip would take place at the wheel-treads.

The Author has seen it stated that about 60 years ago cylindrical
tires were adopted on the Vienna–Salzburg line, but that after continuous
trouble, with flanges and with creeping rails on curves, the experiment was
abandoned. Cylindrical tires are now being tried again, on high-speed
trains, both in Great Britain and in the United States.

Coning, with a moderate amount of centering action as heavily damped
as possible, seems to be the ideal. The trouble is that, with wear, the
centering action becomes excessive. There is surely no way of preventing
tires and rails wearing to fit one another. The tires of passenger vehicles
are sometimes turned to the composite profile shown in *Fig. 20*, in order
to increase the amount of wear necessary to make them fit the rails. If
this composite coning were universally adopted, the profile of the rail-
surface would presumably change, becoming flatter or even hollow; the
initial rail-profile is almost immaterial, as it is so soon worn away. Partial
adoption of composite coning gives relief to the favoured wheels, but the
rail-profile must be slightly modified, to the disadvantage of all others.

At the Author's suggestion, special brake-blocks with a circumferential slot were tried, so that brake-wear was concentrated at the inner and outer edges of the tires, where little running wear occurs. It was hoped that every application of the brakes would then do something to machine up the tires and retain the proper profile. The result was disappointing; it suggested that instead of wearing down the unwanted parts, the brake-blocks deposited metal upon them !

Much time was spent with the model in trying to damp out the oscillation. Two devices seemed effective : the reduction of the axlebox-plays to very small amounts, and giving the axles end play only, suitably resisted. The first was not practical, and the second was tried on a full-size bogie but without marked success.

There remains the possibility of revolutionary design, such as a bogie having independently-rotating wheels. Here the question of price arises ; the conventional design is cheap, and added life before the tires need returning is only worth a limited extra cost.

The Author has not found a solution to the problem ; perhaps the data which he has amassed will help someone else to do so.

For bogie vehicles, the model-experiments and the analysis suggest that :—

(i) The ratio of the mass of the bogies to the mass of the body should be as low as possible ; the mass of the body increases wheel-adhesion, whilst at high speeds its lateral accelerations are out of phase with those of the bogies.

(ii) The critical speed should be as low as possible. This means a low natural frequency of oscillation of the coach, and therefore weak lateral control by the suspension of the coach from the bogies. If this control is too weak, the coach will hit the stops on the bogies too often ; the practical limit in this direction may already have been reached.

(iii) Moderate damping in the coach suspension is desirable. The amount provided in practice seems adequate, as there is no sign of an uncomfortable critical speed.

The Paper is accompanied by twenty-one diagrams and seven photographs, from some of which the Figures in the text and Appendixes, and the half-tone page-plate, have been prepared, and by the following Appendixes.

APPENDIX I.

A Single Axle and Pair of Coned Wheels moving Unconstrained.

On the diagrammatic plan in *Fig. 21*, EF and LH are the inner edges of the rails with which the wheels make contact at A and B. *OX* is the centre-line of the track and G, the centre of the axle, is at (x, y). The planes of the wheels make an angle ψ with *OX*; y and ψ are small.

Let the dimensions be: gauge $2b$; mean wheel-radius r; tangent of the coning angle of the tires T; total mass $2M$, giving equal wheel-loads Mg; moment of inertia of the wheels and axle about a vertical axis through G, $2Mk^2$; speed V, assumed constant.

If D is written for $\dfrac{d}{dx}$, then $\dfrac{d^2}{dt^2} = V^2 D^2$.

Fig. 21.
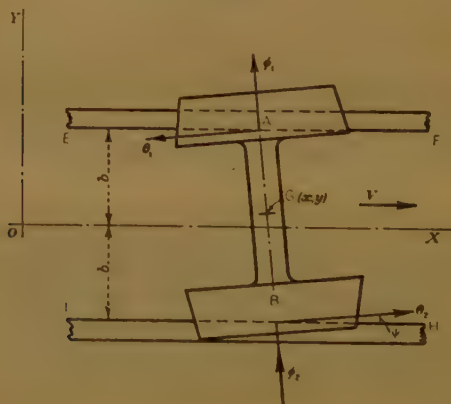
Let the longitudinal and lateral creepages be θ_1, θ_2, and ϕ_1, ϕ_2, respectively, assumed positive in the senses shown. The corresponding forces on the wheel-treads will be $MgK\theta_1$, etc., in the opposite senses.

Since the wheels are rigidly connected, $\phi_1 = \phi_2 = \phi$, say.

Assume that the wheels and axle are propelled by a horizontal force through G parallel to the rails. Since the effective radii of the wheels cannot differ in practice by more than 1 part in 400, then by moments about the axle, with sufficient accuracy,

$$MgK\theta_1 = MgK\theta_2, \text{ or } \theta_1 = \theta_2 = \theta, \text{ say.}$$

Resolving laterally,

$$2M\frac{d^2y}{dt^2} + 2MgK\phi = 0,$$

or, writing

$$\frac{qK}{r^2} = u \text{ and } \frac{d^2y}{dt^2} = V^2D^2y,$$

$$D^2y + u\phi = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

By moments in a horizontal plane,

$$2Mk^2 \frac{d^2\psi}{dt^2} + 2bMgK\theta = 0,$$

or

$$k^2D^2\psi + ub\theta = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

By geometry,

$$dy = \psi dx + \phi\, dx,$$

or

$$Dy = \psi + \phi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Also by geometry, since the effective radii of the wheels are $(r + Ty)$ and $(r - Ty)$,

$$2b.d\psi = 2\theta.dx - 2Ty\,\frac{dx}{r}, \text{ or writing } \frac{T}{rb} = a^2,$$

$$D\psi = \frac{\theta}{b} - a^2y \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Finally, eliminating θ, ϕ and ψ from equations (1)–(4), and writing $p = \dfrac{k^2}{b^2}$,

$$[pD^4 + (1 + p)uD^3 + u^2D^2 + u^2a^2]\,y = 0 \quad . \quad . \quad . \quad . \quad (5)$$

This is the general equation for a single axle.

If V is small, u is great, and the equation becomes

$$[D^2 + a^2]y = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

This crawl-equation for a single axle is well known; it represents a sine-curve of fortuitous amplitude and of wave-length $L = \dfrac{2\pi}{a} = 2\pi\sqrt{\dfrac{rb}{T}}$, or 10·92 feet for the model.

For moderate values of V, the approximate solution of (5) for the motion which persists is

$$y = Ae^{qx} \sin ax, \text{ where } q = \frac{a^2}{2u}(1 + p) \quad . \quad . \quad . \quad . \quad (7)$$

The wave-length is therefore unchanged, but the amplitude of each half oscillation is about $\left(1 + \dfrac{\pi q}{a}\right)$ times that of its predecessor. Taking p as unity ($k = b$), which cannot be far out, this " building-up " factor becomes $\left(1 + \dfrac{2\pi^2}{gK} \cdot \dfrac{V^2}{L}\right)$, or $(1 + 0.00036\,V^2)$ for the model, taking K as 155 and measuring V in feet per second.

For the model, equation (7) should be an accurate enough solution of equation (5) for speeds below about 25 feet per second; above that speed the whole analysis fails, since, in all but the smallest oscillations, $\dfrac{R}{P}$ exceeds 0·05, and K is therefore no longer a constant.

APPENDIX II.

Axlebox and Bearing Clearances in a Four-wheeled Vehicle or Bogie.

Axlebox and bearing clearances impose definite limits on the individual motions of the axles.

Relative to the frame, let each axle have an angular play (in plan) of $\pm\beta$, and an end play (in the direction of its length) of $\pm\delta$. Both β and δ are made up of the plays

of the axleboxes in their guides and of the journals in their bearings. Call the wheel-base $2c$, and put $\dfrac{\delta}{c} = \gamma$; then β and γ are both angles. Assume that, in their middle positions, the axles are parallel and in track. In the diagrammatic plan in *Fig. 22*, the centre of the front axle is at (x_1, y_1) and the front wheels make an angle ψ_1 with OX (the centre-line of the track); the corresponding quantities for the back axle are x_2, y_2, and ψ_2. $G(x, y)$ is the centre of the line joining (x_1, y_1) and (x_2, y_2). The values of y_1, y_2, ψ_1, and ψ_2 are all small.

Suppose that y_1 and ψ_1 are fixed, and consider the limitations on y_2 and ψ_2. If there were no plays then $y_2 = y_1 - 2c\psi_1$; but the angular play, $\pm\beta$, of the frame on the front axle gives y_2 a freedom of $\pm 2c\beta$, and the end plays of the axles a further freedom of $\pm 2\delta = \pm 2c\gamma$. The condition for y_2 is, therefore,

$$|y_1 - y_2 - 2c\psi_1| < 2c(\beta + \gamma) \quad \ldots \quad \ldots \quad \ldots \quad (8)$$

Fig. 22.
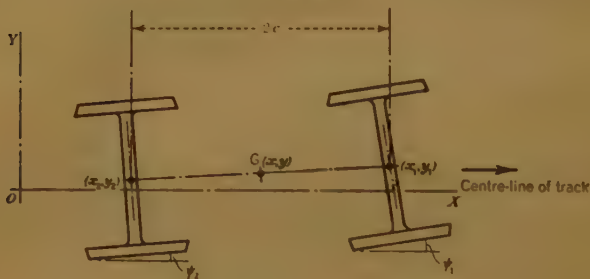
Let the longitudinal axis of the frame make an angle ψ' with OX; then ψ' is subject to two conditions,

$$\left|\psi' - \frac{y_1 - y_2}{2c}\right| < \gamma, \text{ and } |\psi' - \psi_1| < \beta;$$

but ψ_2 is subject to the condition $|\psi' - \psi_2| < \beta$,

whence

$$|y_1 - y_2 - 2c\psi_2| < 2c(\beta + \gamma), \quad \ldots \quad \ldots \quad \ldots \quad (9)$$

and

$$|\psi_1 - \psi_2| < 2\beta \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (10)$$

With sufficient accuracy, at moderate speeds, it may be assumed that the individual axle motions are given by equation (6); hence in terms of x, not of x_1 and x_2,

$$y_1 = A_1 \sin ax,$$
$$y_2 = A_2 \sin (ax + B),$$
$$\psi_1 = \frac{dy_1}{dx} = aA_1 \cos ax,$$
$$\psi_2 = \frac{dy_2}{dx} = aA_2 \cos (ax + B),$$

where A_1, A_2, and B are arbitrary, subject to conditions (8), (9) and (10). Substituting these values in conditions (8) and (9),

$$|A_1 \sin ax - A_2 \sin (ax + B) - 2acA_1 \cos ax| < 2c(\beta + \gamma), \quad \ldots \quad (11)$$

and

$$|A_1 \sin ax - A_2 \sin (ax + B) - 2acA_2 \cos (ax + B)| < 2c(\beta + \gamma). \quad (12)$$

If the left-hand sides of conditions (11) and (12) are represented by vectors, then, as conditions of no interference throughout the cycle,

$$A_1\sqrt{\{m^2 + h^2 - 2mh \cos(B + F)\}} < 2c(\beta + \gamma), \quad \ldots \quad (13)$$

and
$$A_1\sqrt{\{m^2h^2 + 1 - 2mh \cos(B + F)\}} < 2c(\beta + \gamma), \quad \ldots \quad (14)$$

where $m^2 = 1 + 4a^2c^2$ and $\tan F = 2ac$, dimensionless constants of the vehicle, and $h = \dfrac{A_2}{A_1}$.

Further, if A' is the greater of the two amplitudes A_1 and A_2, and $n(<1)$ is their ratio, conditions (13) and (14) can be reduced to a single condition for no interference,

$$A' < \frac{2c(\beta + \gamma)}{\sqrt{\{m^2 + n^2 - 2mn \cos(B + F)\}}} \quad \ldots \quad (15)$$

Now if A is the amplitude of the motion of G, then $2A = A'\sqrt{(1 + n^2 + 2n \cos B)}$; so, substituting for A' in condition (15),

$$A < c(\beta + \gamma)\sqrt{\left\{\frac{1 + n^2 + 2n \cos B}{m^2 + n^2 - 2mn \cos(B + F)}\right\}} \quad \ldots \quad (16)$$

Returning to condition (10), substituting for ψ_1 and ψ_2, and proceeding as before,

$$A' < \frac{2\beta}{a\sqrt{(1 + n^2 - 2n \cos B)}}, \quad \ldots \quad (17)$$

and
$$A < \frac{\beta}{a}\sqrt{\left\{\frac{1 + n^2 + 2n \cos B}{1 + n^2 - 2n \cos B}\right\}} \quad \ldots \quad (18)$$

Provided that A is within the limits fixed by conditions (16) and (18), the axles are free throughout the cycle from positive constraint on their individual motions; the limit fixed by condition (16) is usually the lower. It is shown in Appendix III (D) (p. 259) that, for the model and for a normally-proportioned bogie with unworn tires, control by the axle suspensions should make n tend to unity and B to about 335 degrees. For the model $m = 1\cdot5$ and $F = 48$ degrees, values which would also apply to a normally-proportioned bogie. Inserting these figures in conditions (15)–(18) and writing $a = \dfrac{2\pi}{L}$, gives

$$A' < 2\cdot9c(\beta + \gamma) \text{ and } < 0\cdot73L\beta, \quad \ldots \quad (19)$$
$$A < 2\cdot8c(\beta + \gamma) \text{ and } < 0\cdot72L\beta \quad \ldots \quad (20)$$

Finally, giving L and c their values for the model, the conditions for A become
$$A < 32\cdot5\,(\beta + \gamma) \text{ inches and } < 94\beta \text{ inches} \quad \ldots \quad (21)$$

It will be remembered that, strictly, A refers to the motion of G, the mid-point of the line joining the centres of the axles, not to the motion of the centre of the frame. The first condition of (21), which is usually the limiting one, assumes that the end plays δ are fully used, so under this condition the frame has no further lateral freedom relative to G. If β is small and γ large, the second condition of (21) operates, and the centre of the frame has freedom to wander to either side of G, but this case has little practical importance, and A may be taken as referring to the centre of the frame—the stylus of the model.

APPENDIX III.

Four-Wheeled Vehicle with Elastic Axle Suspensions, at a Crawl.

Suppose that the clearances are sufficient to avoid any positive constraint of the axles, but that the movements of the axles relative to the frame are resisted by elastic suspensions. Assume that the wheel-loads are equal.

On the diagrammatic plan in *Fig. 23*, the centre of the front axle is at (x_1, y_1) and the front wheels make an angle ψ_1 with OX, the centre-line of the track. The corresponding quantities for the back axle are x_2, y_2, and ψ_2. The line joining (x_1, y_1) and (x_2, y_2) makes an angle ψ with OX, and its centre G is at (x, y).

So
$$y_1 + y_2 = 2y, \quad \ldots \ldots \ldots \ldots \quad (22)$$
and
$$y_1 - y_2 = 2c\phi. \quad \ldots \ldots \ldots \ldots \quad (23)$$

Since creepages are proportional to forces, they are subject to the ordinary conditions of equilibrium. Let longitudinal creepages (θ) and lateral creepages (ϕ) be taken as positive in the senses of the arrows in *Fig. 23 ;* then, by moments for each

Fig. 23.
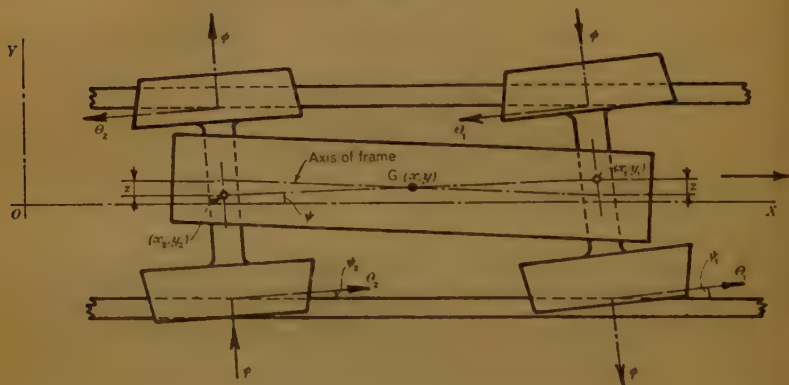
axle, and by moments and resolution for the whole vehicle, they will be related as shown in *Fig. 23*, and

$$b(\theta_1 + \theta_2) = 2c\phi \quad \ldots \ldots \ldots \ldots \quad (24)$$

By geometry, writing D for $\dfrac{d}{dx}$,

$$Dy_1 = \psi_1 - \phi, \quad \ldots \ldots \ldots \ldots \quad (25)$$

$$Dy_2 = \psi_2 + \phi, \quad \ldots \ldots \ldots \ldots \quad (26)$$

$$bD\psi_1 = \theta_1 - ba^2 y_1, \quad \ldots \ldots \ldots \ldots \quad (27)$$

$$bD\psi_2 = \theta_2 - ba^2 y_2 \quad \ldots \ldots \ldots \ldots \quad (28)$$

Let the "equilibrium" position of an axle relative to the frame be defined as the position which the suspension tends to make it take up. Let the axis of the frame be defined as the line joining the centres of the axles in their equilibrium positions. Let these equilibrium positions make angles $\left(\dfrac{\pi}{2} + \alpha_1\right)$ and $\left(\dfrac{\pi}{2} + \alpha_2\right)$ respectively with the axis of the frame. In their equilibrium positions the axles will then be out of parallel

by an angle $(\alpha_1 - \alpha_2) = 2\alpha$, say, converging in the positive direction of OY if α is positive; they will be out of track by a distance $2c\dfrac{\alpha_1 + \alpha_2}{2} = 2c\eta$, say, the front axle being displaced in the negative direction of OY.

Let it be assumed that the suspensions of the two axles are similar, and that any displacement of an axle from its equilibrium position relative to the frame causes, owing to the elasticity of the suspensions, restoring forces on the axle, and therefore creepages by the wheels, proportional to that displacement. If the centre of an axle is displaced a distance z laterally off the axis of the frame, let the restoring creepage be Sz. If an axle is displaced through an angle ξ from its equilibrium position relative to the frame, let the restoring longitudinal creepage of each wheel be $sb\xi$.

Since there can be no lateral force on the frame, the lateral displacements of the axles relative to the frame must be equal and opposite, from which it follows that the axis of the frame always passes through G, which may therefore be regarded as the centre of the frame.

Resolving laterally for either axle,

$$Sz = \phi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (29)$$

The equilibrium position of the front axle makes with OY an angle $\left(\psi + \alpha_1 - \dfrac{z}{c}\right)$,

so
$$\theta_1 = sb\left(\psi - \psi_1 - \frac{z}{c} + \alpha_1\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

Similarly,
$$\theta_2 = sb\left(\psi - \psi_2 - \frac{z}{c} + \alpha_2\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (31)$$

Solving equations (22)–(31) for y, and writing $\dfrac{c^2}{b^2} = f$,

$$[(fS + s)D^4 + s\{(2f + 1)S + s\}D^3 + \{2a^2(fS + s) + (f + 1)s^2S\}D^2$$
$$+ sa^2\{(2f + 1)S + s\}D + a^4(fS + s) + a^2s^2S]y$$

$$= fa^2sS\eta + fs^2S\frac{\alpha}{c}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (32)$$

and
$$\psi = \left[\left(\frac{1}{s} + \frac{1}{fS}\right)D^2 + D + \left(\frac{1}{s} + \frac{1}{fS}\right)a^2\right]y - \eta \quad . \quad . \quad . \quad (33)$$

Some particular cases will be examined.

(A).—Complete angular freedom.

If the suspensions exercise no control over the angular motions of the axles, then $s = 0$. Then, whatever the value of S, equation (32) becomes $(D^2 + a^2)^2y = 0$, or for practical purposes $y = A \sin ax$. This is the equation for a single axle. Equation (33) becomes indeterminate for ψ, showing that the phase-relation between the axle motions is fortuitous.

(B).—Rigid vehicle.

Throughout the cycle, if there are no axlebox or bearing clearances, and during part of any cycle in which the conditions of Appendix II are exceeded, s and S are very great, and equation (33) becomes $Dy = \psi + \eta$ . . . . (34) This represents a crabwise motion due to the axles being out of track, if $\eta \neq 0$.

Equation (32) becomes $\left[(f + 1)D^2 + a^2\right]y = f\dfrac{\alpha}{c},$ . . . . (35)

giving a sine-curve of wave-length $\dfrac{2\pi}{a}\sqrt{f + 1}$. This is called the " long " wave, in

17

distinction from the " short " wave of a free axle. The axles being 2α out of parallel, the axis of motion is displaced $\dfrac{f}{a^2} \cdot \dfrac{\alpha}{c} = \dfrac{c^2}{b^2} \cdot \dfrac{rb}{T} \cdot \dfrac{\alpha}{c} = \dfrac{cr\alpha}{bT}$ in the direction of their convergence. For the model $f = 4$, making the long wave-length $\sqrt{5}$ times the short, or 24·42 feet.

The motion of a rigid four-wheeled vehicle can also be calculated on the assumption of solid-body friction between wheels and rails. The result is practically the same as that obtained here on the assumption of elastic creepage.

Fig. 24.
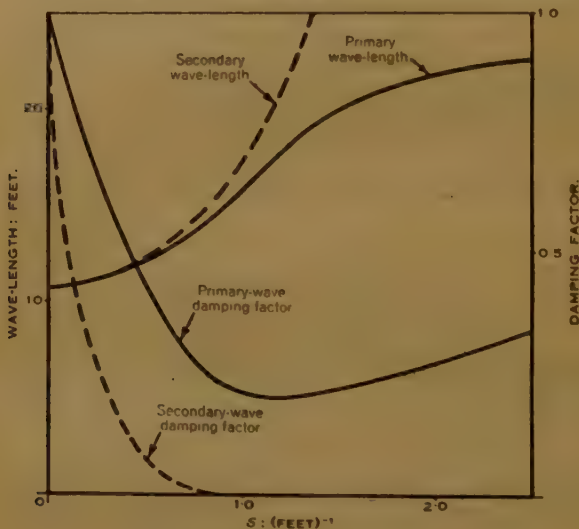
(*C*).—*Effect of varying the stiffness of the suspensions.*

First, let the axles have angular freedom only, so that S is infinite. Then, assuming $\alpha = 0$ and $\eta = 0$ for simplicity, equation (32) becomes

$$[fD^4 + (2f + 1)sD^3 + \{2fa^2 + (f + 1)s^2\}D^2$$
$$+ (2f + 1) sa^2D + fa^4 + s^2a^2]y = 0 \quad . \quad (36)$$

The curves in *Fig. 24* were obtained by giving f and a their values for the model and solving equation (36) numerically for various values of s. They show the relation between the suspension-stiffness s, the wave-length, and the damping factor, the last being the ratio of the amplitudes of successive half-oscillations. If s is small there are two waves: the secondary, specified by the dotted curves, is more heavily damped than the primary. As s increases, the primary wave-length gradually changes from short to long.

Next, let the axles have both lateral and angular freedom, and let $S = 2.25s$, which is about the ratio of the quantities in the model. Then proceeding as before, curves are obtained which differ only very slightly from those in *Fig. 24.*

If s and S for the model depend only on the inclinations of the swing-links, their values are about 0·015 and 0·033 (feet)$^{-1}$ respectively. *Fig. 24* shows that these values are too small to cause much damping.

Finally, let the axles have end (lateral) play only. Then s is infinite and equation (32) becomes

$$[D^3 + (f+1)SD^2 + a^2D + a^2S]y = \frac{fS\alpha}{c} . \quad . \quad . \quad . \quad . \quad . \quad . \quad (37)$$

If S is less than about 0·05 (feet)$^{-1}$, a sufficiently accurate solution is

$$y = Ae^{-fSx/2} \sin ax + Be^{-Sx} + \frac{f\alpha}{ca^2},$$

Fig. 25.
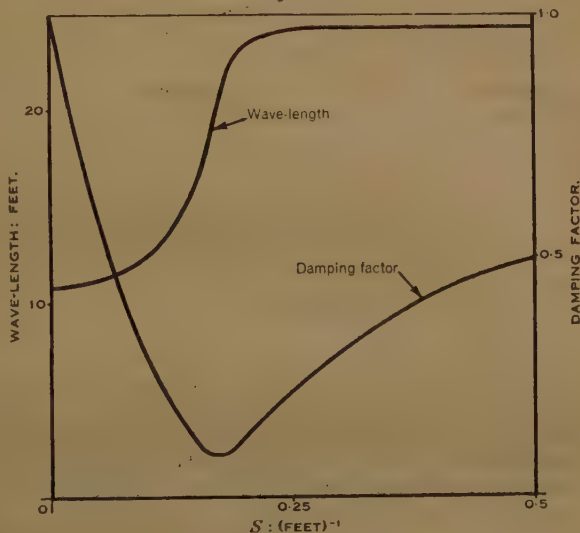
showing that the oscillation is damped and that, if the axles are not parallel, the axis of motion is displaced, just as with a rigid vehicle. More exactly, inserting figures for the model in equation (37), and solving for a series of values of S, the curves in *Fig. 25* are obtained. As S is increased, the wave-length changes from short to long; when S is about 0·17 (feet)$^{-1}$ the damping is very heavy.

(D).—*Relative amplitudes and phase of the axle motions.*

From equations (22) and (23),

$$y_1 = y + c\psi . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (38)$$

and

$$y_2 = y - c\psi . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (39)$$

From equation (33), putting $\eta = 0$ for simplicity,

$$\psi = Dy + \left(\frac{1}{s} + \frac{1}{fS}\right)(D^2 + a^2)y \quad . \quad . \quad . \quad . \quad . \quad (40)$$

Take first the case of angular play only (S infinite), and assume that s is less than about 0·05 (feet)$^{-1}$. By numerical solution of equation (36) for the model, the primary wave is approximately $y = Ae^{-0·305sx} \sin ax$ . . . . (41)

Then, with sufficient accuracy, after the disappearance of the secondary wave, which is much more heavily damped than the primary, equations (38)–(41) give the vector-

diagram in *Fig. 26*. This shows that the axle motions tend to equality in amplitude with the front axle leading in phase by 25 degrees.

Fig. 26.
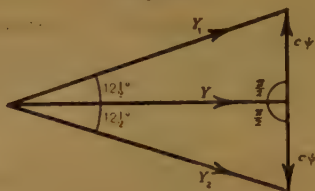
The assumption that $S = 2 \cdot 25s$, s still being less than $0 \cdot 05$ (feet)$^{-1}$, leads to a vector-diagram which is almost identical.

APPENDIX IV.

Four-wheeled Vehicle at Speed.

At speed dynamic effects must be considered, and the analysis is therefore more complicated.

If the axles have lateral and angular freedom, the initial values of twelve quantities are independently arbitrary. These quantities are the lateral and angular displacements and velocities of both axles and of the frame. The equation of motion is therefore of the twelfth order; closely written, it fills a quarto page. In view of the assumptions which must be made regarding the nature of the axle suspensions, such laborious mathematics are not justified.

Fortunately there is a particular case which is easily solved owing to its symmetry, and which shows well enough the character of the motion. Suppose that the axles have unresisted angular, but no lateral, freedom.

In the diagrammatic plan in *Fig. 27*, the centre of the front axle is at (x_1, y_1) and the planes of the front wheels make an angle ψ_1 with OX, the centre-line of the track; the corresponding quantities for the back axle are x_2, y_2, and ψ_2. The line joining (x_1, y_1) and (x_2, y_2) is fixed to the frame and makes an angle ψ with OX; its centre G (x, y) is a point fixed to the frame. The creepages θ_1, θ_2, ϕ_1, and ϕ_2 are assumed positive in the senses shown. The wheelbase, gauge, and other dimensions are as before. The total mass of the vehicle is $4M$, giving equal wheel-loads Mg; the moment of inertia of an axle and pair of wheels about a vertical axis through the centre of the axle is $2M'k^2$. Let $p = \dfrac{M'k^2}{Mb^2}$ and $u = \dfrac{gK}{V^2}$.

Then for the front axle,

by geometry
$$D\psi_1 + a^2 y_1 = \frac{\theta_1}{b},$$

and by moments
$$pbD^2\psi_1 + u\theta_1 = 0.$$

Eliminating θ_1,
$$(pD^2 + uD)\psi_1 + ua^2 y_1 = 0 \quad . \quad . \quad . \quad . \quad . \quad (42)$$

Similarly, for the back axle
$$(pD^2 + uD)\psi_2 + ua^2 y_2 = 0 \quad . \quad . \quad . \quad . \quad (43)$$

From equations (42) and (43), since $y_1 + y_2 = 2y$,

$$(pD^2 + uD)(\psi_1 + \psi_2) + 2ua^2y = 0 \quad . \quad . \quad . \quad . \quad (44)$$

For the whole vehicle,

by geometry $\quad\quad\quad 2Dy = (\psi_1 + \psi_2) + (\phi_1 + \phi_2), \quad . \quad . \quad . \quad . \quad . \quad (45)$

and resolving laterally, $\quad 2D^2y + u(\phi_1 + \phi_2) = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (46)$

Fig. 27.
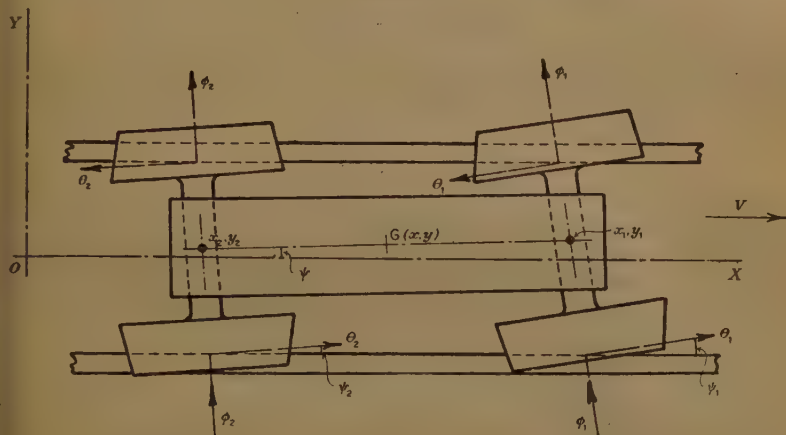
Eliminating $(\psi_1 + \psi_2)$ and $(\phi_1 + \phi_2)$ from equations (44)–(46) gives the equation of motion,

$$[pD^4 + (1 + p)uD^3 + u^2D^2 + u^2a^2]y = 0 \quad . \quad . \quad . \quad (47)$$

This is equation (5) over again, except that p stands for a different ratio, so that the approximate solution for moderate speeds is

$$y = Ae^{qx} \sin \, ax, \text{ where } q = \frac{a^2(1 + p)}{2u}.$$

The wave-length L is still $\dfrac{2\pi}{a}$, and the amplitude of each half oscillation is approximately $\left\{ 1 + \dfrac{\pi^2 V^2(1 + p)}{gKL} \right\}$ times that of its predecessor. For the model, this building-up factor is $(1 + 0.00024V^2)$, where V is measured in feet per second.

The equation of motion of any point on the frame other than G is of the eighth order, and the wheel-base and the radius of gyration of the frame appear. This is to be expected, since the lateral and angular displacements and velocities of the two axles have independently arbitrary starting values.

Paper No. 5201.

"The Vertical Path of a Wheel Moving Along a Railway Track." †

By Professor CHARLES EDWARD INGLIS, O.B.E., M.A., LL.D., F.R.S., M. Inst. C.E.

TABLE OF CONTENTS.

INTRODUCTION.

THIS mathematical investigation, which has been carried out in conjunction with full-scale experiments, was undertaken to study the manner in which the running of a pair of wheels and axle along a straight railway track is affected by characteristics such as the elastic yielding of the ballast, the stiffness of the rail, the lack of continuity at a rail-joint, wheel-loads, and speed. For a continuous length of rail, the pronounced vertical oscillations which at certain critical speeds are induced when sleepers are uniformly pitched has been investigated, and, in studying the disturbances produced by rail-joints, the merits of long and short fishplates have been compared. The graphs by which the calculations are illustrated show the vertical movement of a wheel as it moves along the track, and, to study dynamic effects, the path of the wheel for each quality of ballast and each type of rail-joint has been computed for four different speeds, namely 0, 30, 60, and 90 miles per hour.

In ascertaining the influence of ballast four different qualities have been considered, ranging from a ballast which is absolutely unyielding to one whose elastic compressibility is abnormally great.

To bring the problem within the scope of mathematical analysis, some preliminary process of idealization was essential. For instance, although not in exact agreement with reality, it has been assumed that the ballast yields in a truly elastic manner, the depression of a sleeper being directly proportional to the load the sleeper is called upon to support. Also, in studying the path of a wheel as it crosses a rail-joint, it has been assumed that the rail is supported by eight sleepers only, four on either side of the joint, the small fraction of the wheel-load which is transmitted to the sleepers yet more remote being disregarded as negligible in magnitude. Furthermore, when computing the paths of the wheel when moving at various speeds, the inertia-effect of the rail has been neglected as small in comparison with the inertia-effect of the much more massive moving load.

Throughout the calculations a uniform pitch of 30 inches has been taken for the sleepers along a continuous length of rail, but for the two sleepers on either side of a rail-joint the distance between centres is reduced to 24 inches where a long fishplate is employed, and to 15 inches in the case of a short fishplate.

The majority of the calculations refer to the British standard 95-lb.-per-yard bull-head rail, whose cross section has a moment of inertia of 36 inch[4] units ; for purposes of comparison, however, computations have been made in respect of a much stiffer hypothetical rail whose cross section has been assumed to have a moment of inertia of 72 inch[4] units. For a rail-joint with an accurately-fitted long fishplate, it was found by experiment that the abrupt change in slope at the joint produced by a given bending moment was the same as the total change in slope along a 24-inch length of continuous rail subjected to terminal bending moments of that same magnitude. For a rail-joint with an accurately fitted short fishplate the corresponding length was found to be 66 inches. For the ballast, the connexion between $2R$, the total weight on a sleeper measured in tons, and y, the depression thus produced measured in inches, is assumed to be of the form $R = py$, and for the four types of ballast considered p has the values infinity, 250, 125 and 62·5. Of these $p = 125$ corresponds to a ballast of normal elasticity ; $p = 250$ and $p = 62·5$ specify ballasts which are abnormally firm and abnormally yielding respectively, whilst $p =$ infinity gives the case where the sleepers are rigidly supported.

Path of a Wheel Crawling Along a Continuous Length of Rail.

For determining the path of a wheel as it crawls slowly along a continuous length of track the problem to be solved is set forth in *Fig. 1* (p. 264). The equation giving the rail-deflection from A to B is :—

$$- c^3 py = R_1 x^3 + R_2 \{x - l\}^3 + R_3 \{x - 2l\}^3 + R_4 \{x - 3l\}^3$$
$$- W\{x - (3l + a)\}^3 - \frac{x}{l}[R_1(l^3 - c^3) + R_2 c^3] - R_1 c^3,$$

where $c^3 = \dfrac{6EI}{p}$, and terms inside { } brackets are omitted if they are

negative in value. This expression for y satisfies the conditions $R_1 = py_1$

when $x = o$, and $R_2 = py_2$ when $x = l$. From the conditions $R_3 = py_3$

Fig. 1.
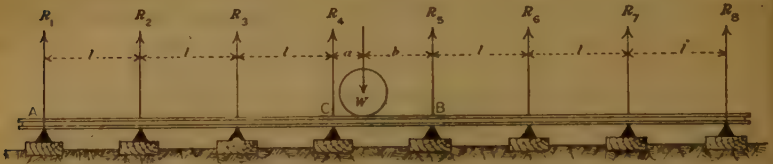
when $x = 2l$, $R_4 = py_4$ when $x = 3l$, and $R_5 = py_5$ when $x = 4l$, R_1,
R_2, and R_3 can be expressed in terms of R_4, R_5, and W. By a precisely
similar process R_8, R_7, and R_6 can be expressed in terms of R_5 and R_4.
Then by stating the conditions of equilibrium for all the vertical forces
acting on the rail, two further equations are obtained, and from these
R_4 and R_5 can be expressed in terms of W, and the corresponding values of

Fig. 2.
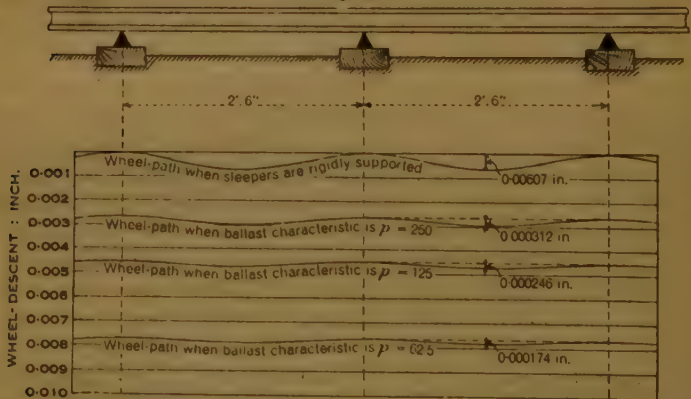
PATH OF WHEEL MOVING SLOWLY ALONG A CONTINUOUS RAIL (MOMENT OF
INERTIA OF RAIL-SECTION 36 INCH⁴ UNITS, LOAD PER RAIL 1 TON).

R_1, R_2, R_3, R_8, R_7, and R_6 then deduced. Having computed the eight
sleeper-reactions, the deflection of the rail at the point where W is applied
can then be determined, and by this means the vertical path of the wheel
as it moves slowly from C to B is ascertained.

The results of these calculations are summarized in *Fig. 2*, from which
it appears that the more yielding the ballast the less is the range of the
vertical movement of the wheel. Thus, whereas when the sleepers are
rigidly supported the range of movement is 0·000607 W inch, with a soft

ballast whose elastic characteristic is defined by $p = 62.5$ the corresponding range of vertical movement is reduced to $0.000174W$ inch.

PATH OF A WHEEL MOVING SLOWLY ACROSS A RAIL-JOINT.

For determining the path of a wheel as it moves slowly across a rail-joint the problem to be solved is set forth in Fig. 3, Plate 1. The equation giving the form of the rail from A to C has the same general form as before, namely :

$$- c^3py = R_1x^3 + R_2\{x - l\}^3 + R_3\{x - 2l\}^3 + R_4 \{x - 3l\}^3$$
$$- W\{x - (2l + a)\}^3 - \frac{x}{l}[R_1(l^3 - c^3) + R_2c^3] - R_1c^3,$$

where $c^3 = \dfrac{6EI}{p}$, and terms inside $\{\ \}$ brackets are omitted if they are negative in value. This expression for y satisfies the conditions $R_1 = py_1$ when $x = 0$, and $R_2 = py_2$ when $x = l$, and from the further conditions $R_3 = py_3$ when $x = 2l$, and $R_4 = py_4$ when $x = 3l$, R_1 and R_2 can be expressed in terms of R_3, R_4, and W. Similarly R_8 and R_7 can be expressed in terms of R_6 and R_5. By stating the conditions of equilibrium for all the vertical forces acting on the rail two more equations are obtained by which R_3 and R_6 can be expressed in terms of R_4, R_5, and W. Finally, R_4 and R_5 are found by two equations stating (1) the condition that the deflection at C is the same for AC and BC, and (2) the relation connecting the bending moment with the discontinuity of slope at C. The results of these somewhat laborious and lengthy calculations are illustrated by Figs. 4, Plate 1, and it will be seen that the more yielding the ballast the greater is the depth of the " pot-hole " into which a wheel will descend at low speed. Thus, whereas with sleepers rigidly supported the depth of the pot-hole is only $0.000901W$ inch or $0.000465W$ inch with long or short fishplates respectively, the corresponding figures with a yielding ballast ($p = 62.5$) are increased to $0.001808W$ inch and $0.001870W$ inch. It has already been seen that for movement along a continuous length of track this comparison of merits is reversed, and that it is the softest ballast which gives the least range in the vertical movement of a wheel as it passes slowly along the rail.

ADVANTAGES OF A STIFFER RAIL.

By the use of a stiffer rail a reduction can be achieved both in the range of the vertical movement of a wheel moving slowly along a continuous length of rail, and in the depth of the pot-hole it creates at a rail-joint. This fact is made evident by *Figs. 5* (p. 266), which compares the wheel-paths for two rails whose cross sections have moments of inertia of 36 inch[4] units and 72 inch[4] units, the ballast-characteristic in both cases having the normal value $p = 125$. It will be seen that for a continuous length of rail the depression of a sleeper as a wheel passes slowly across it is diminished by

nearly 16 per cent. as the result of using a stiffer rail, and that the range of
vertical movement of the wheel is reduced to the very small value of
0·000087 W inch. With the stiffer rail, the depth of the hollow created by a
wheel as it crosses a rail-joint is 0·000904 W inch with a long fishplate and
0·000990 W inch with a short fishplate, both of which values show a con-
siderable improvement over the corresponding figures for the weaker rail.
The foregoing results, however, only relate to "crawl" speeds, and it remains

Figs. 5.

EFFECT PRODUCED ON RAIL-PATHS BY USING A STIFFER RAIL (LOAD PER WHEEL 1 TON),

Full-line curves give paths for a rail whose sectional moment of inertia is 36 inch⁴ units $p = 125$

Dotted "   "   "   "   "   "   "   "   72 " $p = 72$

In both cases the ballast-characteristic is given by $p = 125$

to be seen how far the advantages of a stiffer rail are maintained when the
dynamic effects of speed are taken into account.

DYNAMIC EFFECTS ON THE PATH OF A WHEEL MOVING ALONG A CONTINUOUS LENGTH OF TRACK.

If a pair of wheels and axle standing on a railway track is depressed
below its level of statical equilibrium and is then released, it will perform

vertical oscillations, and the frequency of these oscillations will depend on the mass of the pair of wheels and axle and the elasticity of the ballast and the rail. Usually the elastic yield of the ballast is so great in comparison with the deflection of the rail that the frequency of the vertical oscillations of a pair of wheels and axle is almost independent of its position, and it makes very little difference whether the load is directly over a sleeper or is situated in a midway position. As a wheel moves along a continuous length of track, however, fluctuations of rail-pressure must occur each time a sleeper is traversed, and, if these pressure-variations synchronize with the natural frequency of vertical oscillations, resonance will be established and the consequent vertical oscillations in the wheel and the track may become objectionably violent. The nature of the problem is more precisely set forth in *Fig. 6.*

Fig. 6.
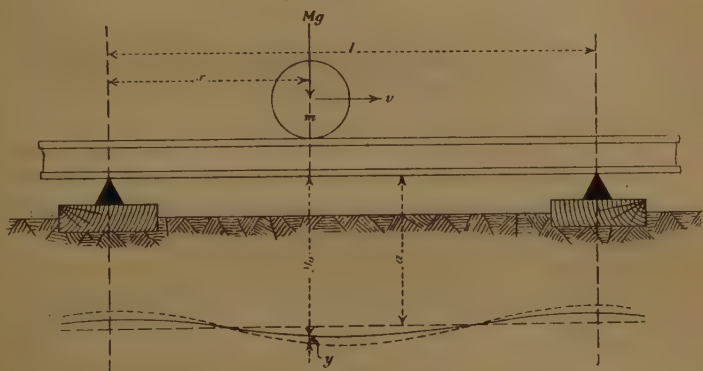
Let m be the unsprung mass per rail, and M the spring-borne mass. Suppose that when the total weight $(M + m)g$ is acting in the position defined by x, the static deflection y_0 produced by this weight is given by $y_0 = a\left(1 - \rho \cos \dfrac{2\pi x}{l}\right)$. Let y be the extra deflection when the speed is v, and let the damping resistance for vertical motion be $K\dfrac{d}{dt}(y + y_0)$. The equation for the vertical movement of m is

$$\frac{d^2y}{dt^2} + \frac{K}{m}\frac{dy}{dt} + \frac{(M + m)y}{my_0}g = -\frac{K}{m}\frac{dy_0}{dt} - \frac{d^2y_0}{dt^2}.$$

Since $y_0 = a\left(1 - \rho \cos\dfrac{2\pi vt}{l}\right) = a(1 - \rho \cos \Omega t)$, where $\Omega = \dfrac{2\pi v}{l}$, then $\dfrac{dy_0}{dt} = a\rho\Omega \sin \Omega t$ and $\dfrac{d^2y}{dt^2} = a\rho\, \Omega^2 \cos \Omega t$. Also $\dfrac{1}{y_0} = \dfrac{1}{a}(1 - \rho \cos \Omega t)^{-1} = \dfrac{1}{a}(1 + \rho \cos \Omega t)$, approximately, if ρ is small.

Hence, writing $\omega_d = \dfrac{K}{m}$, and $\omega_0^2 = \dfrac{M+m}{m} \cdot \dfrac{g}{a}$, the equation for the vertical motion of m takes the form

$$\frac{d^2y}{dt^2} + \omega_d \frac{dy}{dt} + \omega_0^2 y = -a\rho\Omega[\omega_d \sin \Omega t + \Omega \cos \Omega t]$$
$$- y\rho\omega_0^2 \cos \Omega t.$$

The final term on the right-hand side of the above equation is usually of minor importance, and in obtaining the first approximation of y it may be neglected. This first-approximation value of y is then substituted in the term $- y\rho\omega_0^2 \cos \Omega t$, and the solution of the differential equation thus constituted gives the second-approximation value of y. By repeating this process y can be determined to any degree of accuracy; in any particular case, however, the second approximation for y is almost always sufficiently accurate.

The results of applying this method are given in *Fig.* 7. For the wheel-paths plotted, the unsprung mass of a pair of wheels and axle has been taken as 1·8 ton, and the spring-borne load has been assumed to exert a constant downward force of 10 tons. A damping factor of $\frac{1}{2}$ has been taken; in other words, it is assumed that free vertical oscillations die down in a geometric progression whose common ratio is one-half.

The salient points which emerge are set forth in Table I.

TABLE I.

Type of ballast:	$p = 62\cdot5$ (soft).	$p = 125$ (normal).	$p = 250$ (hard).
Resonance speed : miles per hour .	63·8	82·6	106·6
Range of wheel movement : inch .	0·0048	0·0066	0·0083
Maximum acceleration of wheel .	0·32g	0·83g	1·68g

When the sleepers are rigidly supported, the natural frequency for vertical oscillations depend very much on the actual position of the load. As the load approaches a support its frequency becomes indefinitely high, and this extreme variation in natural frequency has the advantage of giving immunity from resonance. Thus in its passage along a rail, if a wheel when midway between two sleepers has somehow or other acquired a free oscillation, then as the wheel approaches a sleeper this free oscillation will become higher and higher in frequency and less and less in amplitude, until by the time the wheel has actually reached the point of support all its energy will have passed into the rail. If the rail, in addition to being rigidly supported at the sleepers, can also be regarded as encastered there, the energy put into any one bay of the rail will not be transmitted to the

next. Thus each time the wheel passes over a sleeper it is completely deprived of vertical movement; such kinetic energy as it may have acquired during its passage from one sleeper to the next it leaves behind,

Fig. 7.
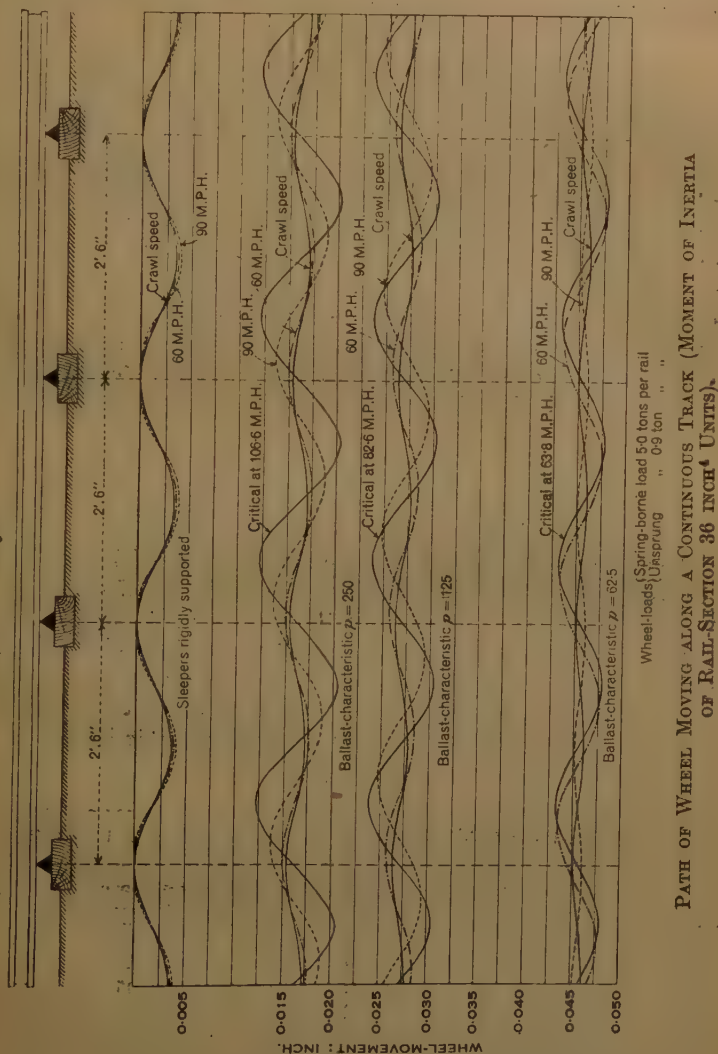
PATH OF WHEEL MOVING ALONG A CONTINUOUS TRACK (MOMENT OF INERTIA OF RAIL-SECTION 36 INCH⁴ UNITS).

and resonance, which consists of the progressive building up of energy, cannot be established.

As is shown in *Fig. 7*, when the sleepers are rigidly supported dynamic effects, even at high speeds, are insignificant, so much so that the paths of

the wheel at 90 m.p.h. and at a " crawl " speed are barely distinguishable from each other.

COMPARATIVE MERITS OF HARD AND SOFT BALLAST FOR CONTINUOUS TRACK.

Leaving out of consideration the abnormal case of rigidly-supported sleepers, the comparative merits of hard and soft ballasts are somewhat conflicting. A soft ballast at most speeds will give less up-and-down motion to a wheel and less vertical acceleration, but there is one critical speed at which the undulatory movement will be considerably amplified, and when the ballast is more than ordinarily compressible, say $p = 62.5$, this critical speed is well within the normal practicable range. On the other hand, with an exceptionally incompressible ballast, say $p = 250$, although the general smoothness of running will not be so good as that given by a soft ballast, the critical speed at which vertical movement will be particularly accentuated is so high that it will seldom, if ever, be attained under normal working conditions.

ADVANTAGES OF A STIFFER RAIL IN CONTINUOUS TRACK.

For a continuous length of rail, both an all-round improvement in the general smoothness of running, and the raising of the critical speed to an exceptionally high figure, can be achieved by using a stiffer rail. This is made apparent in *Fig. 8*, which gives the path of a wheel along a stiffer rail (moment of inertia 72 inch⁴ units), and contrasts it with the path of the same wheel along the standard rail (moment of inertia 36 inch⁴ units), the compressibility of the ballast ($p = 125$) being the same in each case.

The salient points which emerge are set forth in Table II.

TABLE II.

Standard rail : I = 36 inch⁴ units.			Stiffer rail: I = 72 inch⁴ units.		
Speed : miles per hour.	Range of move-ment : inch.	Acceleration.	Speed : miles per hour.	Range of move-ment : inch.	Acceleration.
Crawl	0·0014	0	Crawl	0·0005	0
30	0·0015	0·02g	30	0·0005	0·01g
60	0·0028	0·21g	60	0·0009	0·07g
90	0·0046	0·70g	90	0·0022	0·32g
Critical speed 82·6	0·0065	0·80g	Critical speed 90·2	0·0022	0·32g

From the results here displayed it is apparent that the stiffer rail can

claim an all-round advantage. The critical speed is raised from 82·6 to 90·2 miles per hour, and, even at this enhanced speed, the amplitude and the acceleration of the vertical motion stimulated thereby are con-

Fig. 8.
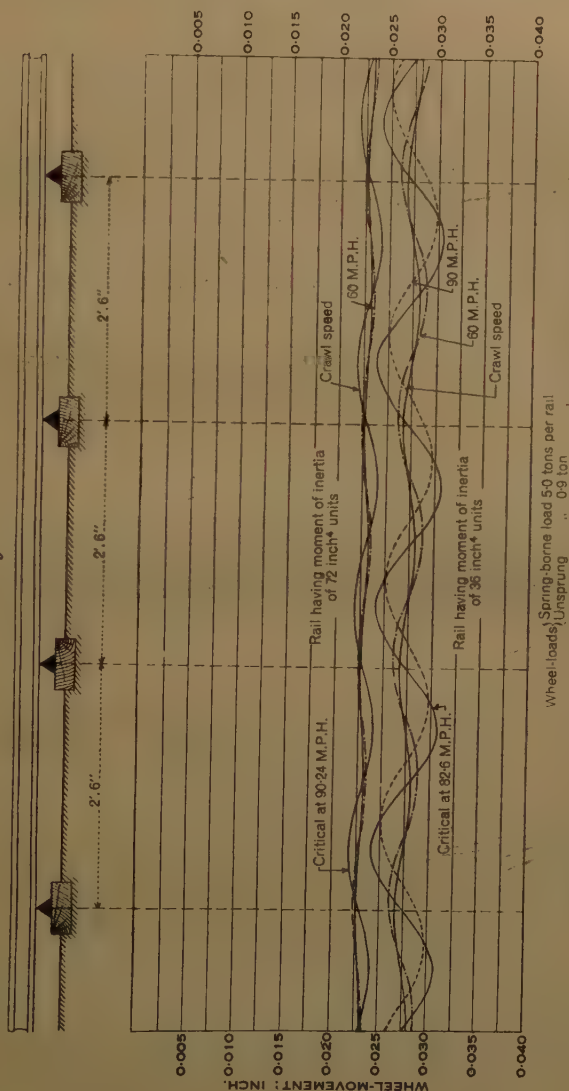
REDUCTION IN VERTICAL MOVEMENT EFFECTED BY USING A STIFFER RAIL.

siderably less than those induced by the lower critical speed associated with the standard rail. At the lower speeds of 60 and 30 miles per hour the reductions in the amplitude and acceleration of the vertical movement of

the wheel are even more pronounced. A further merit which can be claimed by the stiffer rail is that it distributes the load more evenly amongst the sleepers, and consequently a reduction in the pounding to which the ballast is subjected may be expected.

DYNAMIC EFFECTS AT A RAIL-JOINT.

The general nature of these dynamic effects is indicated by *Fig. 9.* The curve $A_0B_0D_0$ shows the path of a wheel as it moves slowly across a rail-joint. When travelling at speed, if the wheel followed this same path, then for the portion A_0B_0, as well as for the portion B_0D_0, since both of these have their concavities facing downwards, the wheel will have a downward acceleration. This downward acceleration reduces the rail-pressure, and

Fig. 9.
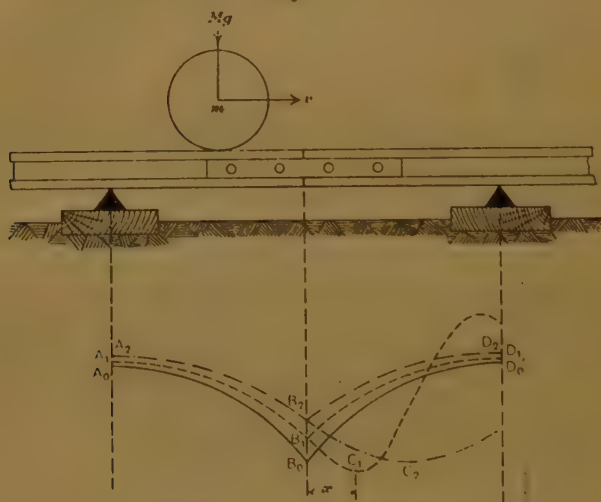
consequently instead of following the curve $A_0B_0D_0$, the wheel-path will be raised to a new formation $A_1B_1D_1$, which can be calculated by a process described later on. This modified wheel-path, however, requires a yet more drastic rectification. At speed, the abrupt change in slope at B_1 would necessitate an abrupt change in the vertical velocity of the wheel, and such a discontinuity in velocity is quite impossible. To preserve continuity in the slope of the wheel-path as it crosses the rail-joint, a free oscillation is brought into existence, and, after passing the joint, the wheel-path is along the curve B_1C_1, a free oscillation, whose frequency can be determined, being superposed on the curve B_1D_1 in such a way that the

discontinuity at B_1 between the slopes of curves A_1B_1 and B_1D_1 is entirely smoothed away.

The frequency of this superposed free oscillation depends upon the elasticity of the track and upon the magnitude of the unsprung part of the moving load, but not on its speed. In consequence the dimension x shown in *Fig. 9*, which defines the position of maximum wheel-descent, approximating as it does to the distance covered in a quarter-period of a free oscillation, will increase very nearly in proportion to the velocity.

If now the speed is increased, the wheel-path will take a new formation $A_2B_2C_2$, as indicated in *Fig. 9*. For the case shown, raising the speed has reduced the depth of the pot-hole into which the wheel descends, and, as the speed is yet further increased, the hollow into which the wheel-descends will become shallower in depth and more extended in length. *Fig. 9* clearly indicates that for some speed, certainly below that which gives the path $A_2B_2C_2$, and possibly even below that which gives the path $A_1B_1C_1$, there is a particular speed for which the wheel-descent is a maximum, and that for higher speeds rail-joint impact becomes progressively less objectionable.

In Figs. 10, Plate 1, a number of wheel-paths for various joints, ballasts, and speeds have been plotted, and from these it appears that the critical speed which gives the maximum wheel-descent for any particular case is by no means sharply defined. The descent of a wheel as it passes over any particular rail-joint varies but little for a considerable range of speed, although ultimately as the speed increases the hollow into which the wheel descends will become progressively less pronounced.

The wheel-paths recorded in Figs. 10, Plate 1, show that, whereas the depth of the pot-hole into which a wheel descends immediately after crossing a rail-joint decreases as the ballast is made more unyielding, the sides of the pot-hole become steeper, and this steepness produces high vertical accelerations. Thus for a really soft ballast ($p = 62\cdot5$), the maximum vertical acceleration of a wheel, whether it is crossing a rail-joint fitted with a long or with a short fishplate, will hardly exceed $2\cdot5g$ at any speed ; but, with an incompressible ballast, although the vertical descent of the wheel is much reduced, the accelerations at high speeds may mount up to as much as $10g$, and perhaps even more.

Fig. 11 (p. 274) indicates the method of calculating how much the crawl-speed path $A_0B_0D_0$ shown in *Fig. 9* must be modified to take into account the reduction in rail-pressure due to speed.

Let M denote the spring-borne part of the moving load, and m the unsprung mass per rail. Suppose that the crawl-deflection under the weight $(M + m)g$ is given by $y = a - bx + cx^2 - dx^3$.

If the speed is v and it is assumed that the wheel follows the above path, its depth at time t after passing the origin is given by $y = a - bvt + cv^2t^2 - dv^3t^3$, and the vertical acceleration when at a

18

distance x from the origin is $v^2 [2c - 6dx]$. The reduction in wheel-pressure is accordingly $(2c - 6dx)mv^2$, and the deflection under the load which takes into account this reduction of pressure is given by

$$y = (a - bx + cx^2 - dx^3) \left[1 - (2c - 6dx) \frac{mv^2}{(M + m)g} \right].$$

Fig. 11.
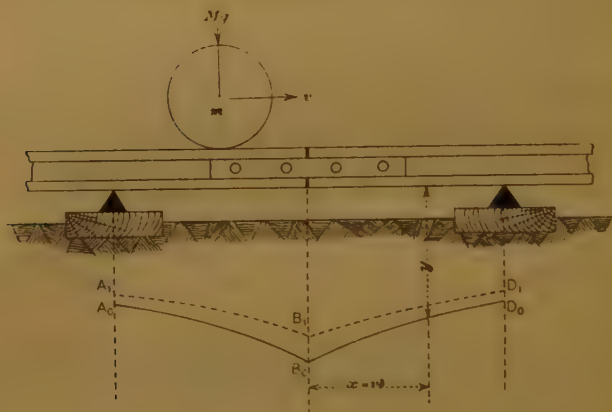
This may be regarded as the first approximation to the wheel-path corrected for inertia, and, if it is not considered sufficiently accurate, it can be used to determine a second approximation by a repetition of the above process.

ADVANTAGES OF A STIFFER RAIL IN REDUCING RAIL-JOINT IMPACT.

All the advantages of a yielding ballast, without any of its disadvantages, can be achieved by using a stiffer rail. This fact is revealed by *Figs. 12*, which give the paths of a wheel over joints in a stiffer rail (moment of inertia 72 inch⁴ units), and compare them with the corresponding paths for a standard rail (moment of inertia 36 inch⁴ units), the compressibility of the ballast ($p = 125$) being the same in each case. From these curves, the results of which are set out in Table III (p. 276), it will be seen that a marked reduction in the wheel-descent is achieved at all speeds, and the curves recorded in *Figs. 13* (p. 276) show that the stiffer rail has also a general beneficial effect in reducing the vertical accelerations produced by a rail-joint.

In the case of a short-fishplate joint the superiority of a stiffer rail in minimizing vertical accelerations disappears at high speeds, but with a stiffer rail it is most unlikely that a short fishplate would be employed.

Figs. 12.
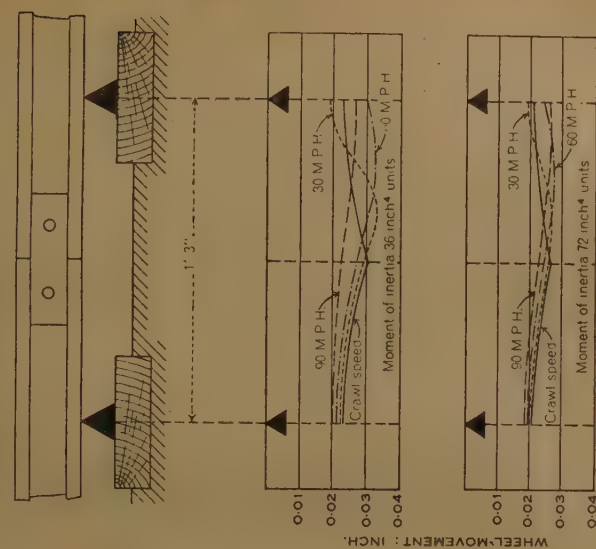
PATH OF WHEEL CROSSING A RAIL-JOINT AT VARIOUS SPEEDS, WITH STANDARD AND STIFFER RAILS ($p = 125$).

TABLE III.

| Standard rail: $I = 36$ inch⁴ units. | | | Stiffer rail: $I = 72$ inch⁴ units. | | |
| Speed: miles per hour. | Wheel-descent: inch. | | Speed: miles per hour. | Wheel-descent: inch. | |
	Long fish-plate.	Short fish-plate.		Long fish-plate.	Short fish-plate.
Crawl	0·034	0·031	Crawl	0·028	0·026
30	0·036	0·034	30	0·028	0·027
60	0·037	0·033	60	0·028	0·028
90	0·040	0·027	90	0·030	0·026

Figs. 13.

LONG FISHPLATE.

SHORT FISHPLATE.

Wheel-loads $\begin{cases}$ Spring-borne load 5·0 tons per rail $\\$ Unsprung   ,,   0·9 ton   ,,   ,, $\end{cases}$

The full-line curves refer to a rail having a moment of inertia of 36 inch⁴ units

,,   ,,   ,,   ,,   ,,   ,,   ,,   72   ,,   ,,

VERTICAL ACCELERATION OF A WHEEL CROSSING A RAIL-JOINT ($p = 125$).

EXPERIMENTAL VERIFICATION OF THEORY.

The foregoing conclusions, relating to the general effects produced by altering track-characteristics and the benefits which may thus be attained, can only command confidence if the results which emerge from analysis stand the test of experimental verification. In this case a systematic experimental research, which has been carried out in conjunction with the

mathematical investigation, provides such evidence in abundant measure, and it is satisfactory to find that wherever theory and experiment can be compared they appear to be in most harmonious agreement. For instance, the theoretical conclusion that the vertical movements of a wheel due to sleeper action become increasingly noticeable at high speeds, whereas those due to rail-joints tend to diminish, is clearly indicated both by vibrograph and accelerometer records. The theoretical computations in general lead to values of accelerations appreciably lower than the highest recorded by experiment, but this is to be expected, since the theoretical calculations relate to a rail which is in perfect condition, whereas the highest accelerations experimentally recorded are always associated with rail-joints having an unfavourable profile, resulting from permanent set. Without adding greatly to the complications of analysis, the impact-effects of a rail-joint having any given profile can be computed, and if an unfavourable profile is assumed, the large vertical accelerations sometimes revealed by experiment can be theoretically reproduced. For drawing general conclusions regarding the effects of different qualities of ballast, the comparative merits of long and short fishplates, and the advantages consequent on using a stiffer rail, it is necessary for the sake of comparison to assume a standard condition for rail-joints, and it is the impossibility of achieving this uniformity which makes it so difficult to deduce general conclusions from experimental observations. The all-important use of experiment is to confirm or to disprove the findings of theory, and, if satisfactory corroboration is forthcoming, .it can then be left to theory to produce generalizations. That policy of experimental verification has been followed in this case, and the fact that such corroboration has been forthcoming is the main justification for putting on record this particular mathematical investigation.

The Paper is accompanied by thirteen sheets of drawings, from which Plate 1 and the Figures in the text have been prepared.

Discussion.

Dr. Davies showed a number of lantern slides illustrating his Paper.

Professor Inglis showed some lantern-slides in illustration of his Paper.

Mr. W. K. Wallace was particularly interested to learn that it was possible to obtain better riding combined with easier maintenance by using a stiffer rail. The London Midland and Scottish Railway would shortly have lengths of main-line track laid with three rail-sections ; namely, the standard 95-lb.-per-yard bull-head rail (moment of inertia 36 inch^4 units), 110-lb.-per-yard flat-bottom rail (moment of inertia 57·2 inch^4 units), and 131-lb.-per-yard flat-bottom rail (moment of inertia 88·5 inch^4 units). In addition to the advantages of a stiffer rail, as compared with the British standard rail, which were pointed out by Professor Inglis, a further advantage was that the stiffer rails, being deeper, enabled heavier fishplates to be used. Whereas the moment of inertia of a pair of fishplates in the 95-lb. rail was only 5·9 inch^4 units (17 per cent. of the moment of the rail), in the case of the 110-lb. rail it was 12·2 inch^4 units (21·4 per cent.), and in the case of the 131-lb. rail it was 32·2 inch^4 units (36·4 per cent.), so that a much more efficient joint was obtained with the heavier rails. The L.M.S. Railway used short fishplates for chaired track, not so much from a desire to save steel—although that saving was not to be despised— as from a desire to get the joint-supports closer together. With flat-bottomed rail, however, it was possible to use long fishplates with closely spaced sleepers. He hoped that the reduction in maintenance mentioned by Professor Inglis would be obtained, and that it would be sufficient to offset the extra cost of heavier rails.

The effect of the ballast was interesting, but in practice he thought it would be necessary to retain the standard stone ballast irrespective of its resistance, because it was the only material with which the packing could be maintained for a sufficiently long time. Even if it were desired to use a softer ballast, it would be impossible to bear the cost of the extra staff then necessary to maintain the track.

In regard to Dr. Davies's Paper, Mr. Wallace thought that there could be no doubt that very much steadier riding was being obtained by alterations in the coning. The research would, he hoped, be carried further, because relative wear between rail and tire was a very real problem ; that, together with the research of Professor Inglis into the vertical oscillations, should do a very great deal to give more comfortable riding, which was, with due regard to economy and maintenance, the desideratum which the railways were striving to attain.

Professor Miles Walker desired in the first place to refer to *Fig. 9* (p. 272) of Professor Inglis's Paper, which showed that concussion occurred at a rail-joint. That diagram was drawn for a perfectly-fitted fishplate, and there was no shearing action shown at all between the ends of the rails. In practice, however, there was a distinct shearing action at an average rail-joint, the relative movement of the two rail-ends often amounting to about 0·05 inch. The wheel-motion thus induced was much more important than that due to the bending of the rail shown in *Fig. 9,* and the fact that that large motion occurred caused the actual vibration of the carriage, which was what the passenger felt. Presumably Professor Inglis would agree that his diagrams were diagrams of the motion of the axle, but the passenger was concerned with the motion of the carriage. The carriage was carried on springs, and had a very much slower period of oscillation than the ballast. For a speed of 60 miles per hour the frequency of oscillation of the ballast, although dependent on the spacing of the sleepers, would be about 35 per second; that was an exceedingly rapid vibration, and the amount of it transmitted to the carriage was un-important, but the vibration felt by the passenger had a low frequency of about 2 per second for a 45-foot rail. That might become very great due to resonance with the oscillation of the carriage on its springs. The actual motion of the carriage took the form of a curve showing con-siderable oscillation at each rail-joint. The movement was a compound effect produced by the two axles of the bogie, and it was often not exactly a sine-curve because the movement due to one axle was superimposed on that due to the other. At a speed of 14 miles per hour the diagram was quite clear, but at a speed of 45 miles per hour the effects of all the rail-joints merged into one continuous oscillation.

He produced records, taken on an instrument designed for the purpose. One of those, taken at a speed of 14 miles per hour, showed clearly the disturbance produced at each joint of about 0·4 inch vertical movement of the coach. In that case the vibration died out before the next rail-joint was reached. In another record taken at about 32 miles per hour, the position of the joints could clearly be recognized, the disturbance to the coach amounting at some joints to 0·8 inch. That record showed one place where the joint was perfectly made and as rigid as the rail. There there was no vertical motion of the carriage at all. Another record taken at a higher speed showed the vibrations produced at one joint running into those produced at the next. There the carriage moved through a vertical distance of 1·3 inch.

Three conditions were necessary to overcome that defect : the fishplates had to be as stiff as the rail, all relative motion between the parts of the joint had to be eliminated, and the tightening of the joints was bound to put upon the fishplates a permanent bending moment comparable with that in the rail during the passage of a heavy locomotive. About 9 years ago an experimental rail-joint had been evolved, and through the kindness

of the late Sir Henry Fowler, K.B.E., M. Inst. C.E., Professor Walker had
been able to have it installed near Stockport. It embodied very thick
fishplates, with extensions under the rails. After the four main bolts were
tightened four screws drove in wedges which put on the permanent bending
moment. The machining operations were very simple. It was demon-
strated that that joint produced no vertical movement of the carriage
and preserved the ends of the rails. The objection which was raised,
however, was the cost. Even, however, if the cost should work out at
£200 per mile of track—and he did not imagine that it would be as
much as that—that cost was not great when compared with the cost of
re-laying even with standard rails, and was even smaller in comparison
with the cost of relaying with heavier rails.

Mr. W. A. Stanier said that Dr. Davies's mathematical analysis and
examination had confirmed what all railway engineers had been aware of
for a very long time, namely, that if tires were worn hollow bad hunting
resulted. He regretted that Dr. Davies had not described more particularly
the experimental work which he had done in conjunction with the L.M.S.
Railway on the riding of railway rolling stock, and that Dr. Davies had not
given some particulars of the visual results obtained by using a cinemato-
graph camera, which had demonstrated very clearly the movement of a
wheel on the rail. The L.M.S. Railway had tried cylindrical wheels, but,
as Dr. Davies had indicated, there was the difficulty of manufacturing
cylindrical wheels of exactly equal size. That was a shop problem which
would necessitate grinders for railway wheels, which was not always pos-
sible. Dr. Davies had not referred to the wheel-coning of 1 in 100 which
the L.M.S. Railway were at present using; wheels with that coning
appeared to be giving in practice as good results as cylindrical wheels.
The most important consideration of all from the Chief Mechanical
Engineer's point of view, however, had not been referred to : how could
the time be lengthened before the wheel had to go into the shops for re-
turning, if bad riding of the coaches were to be avoided ?

Mr. J. Taylor Thompson proposed to speak only on the Paper by
Professor Inglis, and to deal with only one point in it, namely, the question
of the weight of rail. The Paper dealt only with one axle, and not with a
series of axles, and it was clear that with one load the weight was trans-
mitted through the rail over a series of sleepers ; he had assumed that
one-third of the weight was carried in the middle, tapering out for perhaps
eight sleepers. It was clear that the stiffer the rail the greater was the
distribution, and therefore the less pressure there was on the sleepers
adjoining the load. The rail was perhaps acting over a span of something
like 10 feet, and the stiffer it was the better. That was also the conclusion
which Professor Inglis drew. However, when instead of just one load
there was a long series of equal loads spaced at sleeper-spacing apart, it
seemed clear that each of the sleepers was bound to take the full wheel-
load, and that the relief occurred only towards the ends of the series of

loads, where there were free sleepers to which the rail could transmit some of the load. He suggested that that was true however stiff the rail might be. Unless the rail could act as a beam over a span of something like 30 feet to the unloaded parts of the track, the sleeper-load in the centre of the series of loads would be the full axle-load, irrespective of the weight of the rail. That was an important point of distinction between a single axle and a series of closely-spaced axles. In the first case it was obvious that a stiff rail would improve the track and decrease the deflection, but in the second case the deflection of the track was affected very little by the stiffness of the rail ; what affected it was the resistance of the ballast. In practice a series of loads had to be dealt with.

The American Track Stress Committee, which had been sitting for over 20 years, had dealt with the subject of ballast-resistance, and their report seemed to confirm the point of view that he had been putting forward. The Committee used the symbol u to denote the ballast-resistance, and considered both a 2–8–2 locomotive and the adjacent ends of two eight-wheeled bogie coaches. They considered a 90-lb.-per-yard rail with $u = 1,000, 2000,$ and $4,000,$ and a 152-lb.-per-yard rail with $u = 2,000,$ $4,000,$ and $6,000.$ Their report showed that a 90-lb. rail on a '' 2,000 '' ballast showed almost exactly the same deflection as a 152-lb. rail on a '' 2,000 '' ballast, the deflection being just over 0·2 inch in both cases. A 90-lb. rail with '' 4,000 '' ballast and a 152-lb. rail with '' 4,000 '' ballast also showed approximately the same deflection (0·1 inch). The increase in the ballast-resistance, and not the increase in the strength of the rail, was what determined the degree of depression. The rail had, however, one very important effect, in that the stiffer the rail the smaller were the local depressions. The total depression was the same, but with the 152-lb.-per-yard rail the depression was very much smoother.

That raised the problem of how to decrease track-depression. It seemed clearly necessary to increase the ballast-resistance. *Fig. 1* (p. 282) showed the distribution of pressure from the sleepers themselves. The diagram was taken from the results of the American research work, but he had applied it to English practice with a different kind of track and different spacing of sleepers. The diagram showed lines of equal pressure, there being an intense pressure shown in the middle of the sleeper and less at the sides. By the time a distance below the sleeper was reached equal to the spacing of the sleepers, the pressure had become almost uniform ; in other words, if the ballast could be as deep as the spacing of the sleepers there would be uniform pressure on the formation. With about 5 inches of ballast there was heavy compression underneath the sleepers and no pressure in between, and that created pot-holes under the sleepers, leading to water-collection and all its inherent troubles. It would seem that there were two ways in which it was possible to decrease the depression of the track and to stiffen it. One was to place the sleepers closer together, and the other was to increase the depth of ballast so as to increase the area over which

the load was spread, thus decreasing the unit load and consequently the depression. Mr. Thompson then showed a photograph taken some time ago after a serious washout had occurred. Ash ballast had been used under the track, and the photograph showed how the ash immediately

Fig. 1.
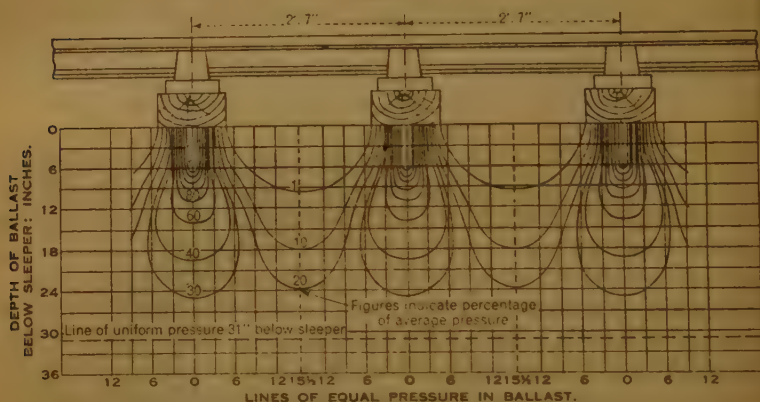
DISTRIBUTION OF PRESSURE FROM SLEEPERS.

below the sleepers had been pressed into a solid mass, whilst elsewhere, under lighter pressure, it had been less compressed and had been washed away.

Finally, he would suggest to Professor Inglis that an ideal subject for further research would be the factor of safety against rail-climbing on curves.

Dr. F. W. Carter observed that, since Dr. Davies had referred to some of his work, there were a few remarks which he might make with regard to it. When he had introduced the creepage coefficient he had had no idea of its magnitude, but afterwards he had found a formula which would, in an ideal case, give a value; that was the formula quoted by Dr. Davies, and he was very glad to see that the figures calculated by his formula and Dr. Davies's experimental results were comparable in magnitude. A Paper had been published in 1935 by the American Society of Mechanical Engineers [1] in which the author stated that the average of a number of actual tests gave a figure of 8·5 million for the creepage coefficient, and that Dr. Carter's formula gave 8·6 million, the difference being less than the probable error of either figure. The American author might have said, however, that the correspondence was fortuitous, because in fact no such exact correspondence

[1] B. S. Cain, "Safe Operation of High-Speed Locomotives." Trans. Am. Soc. C.E., vol. 57 (1935), p. 473.

was to be expected between the two figures. He had not seen his way to calculate the lateral-creepage coefficient, but he had assumed it to be equal to the longitudinal-creepage coefficient. He was glad to see that Dr. Davies had confirmed that it was in fact much the same in magnitude.

He had been interested in the work of Dr. Davies on the effect of clearance in the axleboxes. Dr. Davies showed that that was, at any rate at slow speeds, a factor which tended to damp out the long-wave oscillation. Dr. Carter had made his investigation on the assumption that there were no clearances, and that the truck moved in the way that it was intended to move, the clearance being adventitious although unavoidable. At speed, however, his assumption apparently became justified.

One subject which he had investigated but which Dr. Davies had not considered was the interaction between groups of wheels, in order to see how a locomotive built up, for instance, with a leading bogie, a number of axles in the main frame, and perhaps a trailing (swinging) axle would behave in running. He found that in certain cases the locomotive tended to keep to the middle of the track of its own accord, without the action of the flanges. The question of clearances had not been considered, however.

He considered that the coning of the wheels was not really the cause of instability, but was only a factor which showed it up. He had considered coning rather as a way of bringing the matter within the ambit of mathematics. If there were no coning the motion would have to be unstable, as the truck would always run until a flange struck a rail, to be then deflected against the other; the coning enabled him to investigate the motion, which was sinuous, and he could investigate the question of the stability of the running with that motion.

Dr. W. L. Lowe-Brown felt that Professor Inglis's Paper would have been even more valuable if details had been given of observations on the actual behaviour of the track under the passage of trains. Presumably there would be an appreciable modification of the depression-curves, especially near the joints, caused by the restraining action of the chairs, since the rail, instead of being supported on knife-edges, would be supported almost to the edges of the chairs, so that the effective span would be considerably less than that used in the calculations.

When he was the Chief Engineer of the Buenos Aires Western Railway in 1915, observations had been made of the resilience of various kinds of ballast, its effect on the rail-stresses being worked out by simplified graphical methods on the lines of the crawl-calculations shown by Professor Inglis. The conditions were widely different from those on British railways. The only part of the railway on which stone ballast was used was in the first 50 miles from Buenos Aires, where there was a heavy traffic, including a fast electric service. The remaining 1,800 miles of line

were laid with ashes or selected earth ballast. The rails were 85 lb. or 80 lb. per yard, and flat-bottomed. The deflection of the sleepers under various axle-loads was measured for several different kinds of ballast. The results approximated to those published 4 years later by the American Committee on the Stresses in Rail Road Track[1]. The value of p as used by Professor Inglis would be of the order of from 10 to 20 for earth ballast, and from 20 to 30 for stone ballast. With earth ballast travelling was very much smoother, and rail-wear was definitely less, than with stone ballast. The only difficulty was that with very heavy traffic the cost of maintenance was higher. In one test with 80-lb.-per-yard flat-bottom rails, measurements were taken near a joint of the rail. The joint was formed by two stiff angle fishplates, the ends of which rested on hardwood sleepers at 20-inch centres. The ballast was burned city refuse, which made an excellent track. The maximum axle-load in the test considered was 15·4 tons, and the maximum depression under the inner fishplate-bolt on the " approach " side of the joint was 0·16 inch. That was somewhat less than the average depression at intermediate sleepers at 2-foot 5-inch centres under the same load.

There were many factors which determined the vertical path of a wheel moving along a railway track, the principal ones being the ballast, the sleepers, and the rails. No doubt with the object of simplifying the analysis, Professor Inglis had studied only the variations in the ballast and the rail, and had kept the sleeper-spacing uniformly 2 feet 6 inches, except at the joints. In that way he had arrived at the conclusion that a great improvement could be secured by using a very much heavier rail. He had apparently rejected the other solution of the problem ; namely, the use of additional sleepers to reduce the sleeper-spacing.

There were actually two opposite schools of thought, one considering that heavy rails and a small number of sleepers gave the most economical track, and the other, to which he belonged, preferring lighter rails and a greater number of sleepers. Before the introduction of shovel-packing it might have been considered that with a track of bull-head rails and chairs the minimum distance apart of 10-inch sleepers was 2 feet 6 inches, with something less at the joints, but with shovel-packing he could see no reason why the sleeper-spacing should not be reduced to 20 inches in the middle of the rail and to less than 15 inches at the joint. In Canada and in the United States, where, compared with British practice, very much heavier axle-loads were the rule, not only were heavier rails used, but the sleepers were placed very much closer together. Had Professor Inglis examined the possibility of achieving the same result as he had obtained, by leaving the weight of rail unaltered and reducing the sleeper-spacing ? To obtain the best result it would probably be necessary to reduce the spacing at the joint to less than 15 inches.

[1] Trans. Am. Soc. C.E., vol. 82 (1918), p. 1191.

Mr. W. E. Gelson found it easy to appreciate the difficulties which arose in connexion with the study of nosing of vehicles, as he had recently been concerned with similar work on locomotives. If accelerometers, preferably of the electric recording type, had been placed direct on the axleboxes of the coach tested by Dr. Davies, in order to record lateral movement, much useful information might have been obtained. Such equipment was easily obtainable, and the advantage of obtaining simultaneous movements or accelerations on one graph was very great. It would enhance the value of the Paper if a diagram of the bogie-suspensions and main dimensions were included, with a statement giving the coefficients of friction at the various rubbing surfaces, if they were known. With regard to the effect of end play in the brasses, his own experience with locomotive oscillations was similar to that of Dr. Davies, namely, that the tighter the clearances the greater the instability. Mr. Gelson had found that, with new tires, there was a marked increase in the wave-length with increase of speed, but he had not made tests with worn tires.

Referring to the section of Professor Inglis's Paper dealing with the continuous rail, he noticed that the sleeper-pressures deduced from the deflections plotted in *Fig. 2* (p. 264) differed widely from those obtained by corresponding calculations based on the continuous-elastic-support theory. For example, for $p = 62 \cdot 5$, the maximum sleeper-load from one rail would be 0·238 times the wheel-load, compared with 0·40 times the wheel-load using the older method. It would be very helpful if one example were worked out in detail and added as a guide to engineers who had in the past based their calculations on the continuous-support theory. Professor Inglis considered that a value of $p = 62 \cdot 5$ denoted an abnormally yielding ballast ; such a value would, however, be regarded as of considerably more than average stiffness for first-class 5-foot 6-inch gauge main-line tracks fully ballasted in India, and half that value would be considered to be a good average figure for such track, laid with 90-lb.-per-yard flat-bottom rail on hardwood sleepers. A large series of determinations recently made gave a value for p of 43 for 90-lb.-per-yard rail on steel-trough sleepers, laid on 24 inches of ballast on brick soling. Timber sleepers themselves contributed considerably to the value of p. Tests had shown that pine or deodar sleepers on an infinitely rigid ballast would give a value p of 150. Reduction of the track-gauge was accompanied, other things being equal, by a reduction in the value of p. In India serious oscillation had not been observed at frequencies corresponding to the sleeper-spacing. That was no doubt due to the greater degree of elasticity in the road-beds.

It had been customary in India during the past 15 years, when calculating rail-stresses, to allow for the effect of the speed of the live load. That effect had lately been examined in some detail, and it had been found that it assumed importance where the road-bed had marked flexibility and where the rails had a rather low moment of inertia. The speed-effect

would, he thought, be noticeable in the case of the L.M.S. Railway track
for a value of p of 62·5, but not for values of p higher than that figure.

He would like to draw attention to one minor point in the Paper. In
the first graph (*Fig. 2*, p. 264) for the condition of rigid supports, the
deflection had been calculated assuming that negative reactions would be
provided by the sleepers in the adjacent bays. He was doubtful, however,
whether such negative reactions could be developed under the 5·9-ton
wheel-load considered in *Fig. 7* (p. 269).

Professor Inglis had concluded that an all-round improvement in
smoothness of running could be achieved by using a stiffer rail. That had
been borne out by the experience in India. The advantages of using
track laid with 90-lb.-per-yard rail, compared with track laid with
75-lb.-per-yard rail, had been shown to be very considerable.

Referring to the problem of the rail-joint, as the vertical path of a wheel
was affected greatly by over-stress in the fishplate, it would have been of
considerable interest had Professor Inglis referred to rail- and fishplate-
stresses. Investigations had shown it to be common practice on some
railways to pack joint- and shoulder-sleepers harder than intermediate
sleepers. That practice had been observed only when the design of the
joint had been unequal to the loads for which the track was intended.
Vertical oscillations of the sprung mass due to hard packing at the joints
were of considerable importance at certain speeds.

*** **Mr. O. F. A. Sandberg** observed that both Professor Inglis's and
Dr. Davies's Papers brought forward subjects of very great interest to all
railway engineers. The prevention of uneven riding and vibration,
whether due to track or wheels, was one of particular importance to-day
in view of increasing speeds and loads.

The design and quality of track and tires was a subject to which his
firm had given very prolonged study. Whilst realizing the importance of
ballasting and number of sleepers, they considered that a heavier rail,
and particularly a stiffer rail, gave very great advantages. It was
desirable that the strength of the joint should approximate as closely as
possible to that of the rail. The plain fishplate for the British 95-lb.-
per-yard bull-head rail was in that respect quite weak, and Mr. Wallace's
figures of the strength of flanged fishplates for his heavier flanged rails
were exceedingly interesting.

The successful introduction of the welding of rail joints would by
degrees reduce the number of fishplate joints, but it seemed very important
that such fishplate joints as remained should be improved, apart from
better support by more closely spaced sleepers or by other means.

It should be borne in mind when calculating the strength and stiffness
of track that loads and stresses were very much increased by such factors

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*** This and the succeeding contribution were submitted in writing.—SEC.
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as imperfectly fitting fishplates and imperfect surface of track, and by wear and deformation of the original sections, particularly due to batter at the joints. To reduce the effect of those factors to a minimum it was of primary importance that the best quality materials should be utilized that could most successfully withstand the worst conditions encountered.

Mr. J. W. Eling Smith inquired whether or not the wear on the tread of a tire (excluding wear upon the brake-blocks) might be considered to be a function of the angle of coning; in other words, would a reduced angle of coning reduce the wear on the tread of the tire?

Dr. Davies, in reply, dealt first with the question, raised by Mr. Stanier, of increasing the time before the wheels had to go into the shops for re-turning. In the conventional design, with wheels fixed to the axles, the trouble was that the tires wore to fit the rail-surfaces, and that when a certain degree of fit had been reached the riding became so bad that re-turning of the tires was necessary. Wear could not be prevented entirely; the production of slower wearing tires, without sacrificing other qualities, was a matter for metallurgists. An abortive attempt to make the brake blocks re-turn the tires in service was described in the Paper. The only complete solution seemed to lie in a design with independently rotating wheels. Tire-wear would then not matter, but the increased initial cost might offset the saving on maintenance.

Dr. Davies was interested by Mr. Eling Smith's suggestion that the rate of wear might be affected by the coning angle; he thought that with too little coning the rate might be increased, since small differences between the wheel-radii and slight curves in the track would not then be looked after by the coning, and would cause creep or slip. Too much coning might also increase the rate of wear, since the acceleration forces at the wheel-treads would be increased by the higher frequency of oscillation, leading again to more creep or slip.

He did not agree with Dr. Carter's suggestion that at speed the effect of the axlebox clearances might be neglected. The accelerometer records with new tires, given in curves (1) and (2) of *Fig. 18* (p. 247) were taken in service at speeds of over 70 miles per hour. The wave-length of the oscillation, about 60 feet, agreed very closely with the calculated value for a single free axle; if clearances had been neglected the calculated value would have been about 130 feet. He agreed with Mr. Gelson that it would have been desirable to record the axlebox accelerations electrically; although more expensive, it would have been better than the " Bowden "-wire apparatus actually employed.

The leading dimensions of the bogie used in the full-size tests were given on p. 241. The bogie-frame suspension consisted of laminated side bearing-springs and rubber blocks, as shown in *Fig. 19* (facing p. 227). The coach was suspended from the bogie frame through inclined swing-links, a spring plank, coil springs, and a bolster.
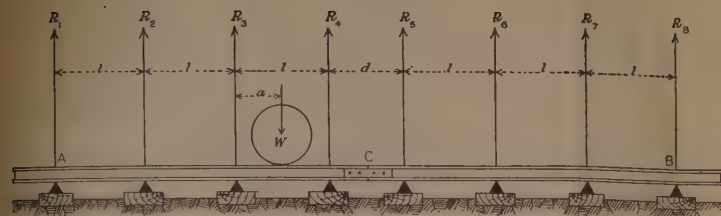
Professor Inglis, in reply, observed that Mr. Thompson called attention to the fact that if a rail carried a number of loads concentrated at fairly close intervals the average downward deflection would not be appreciably reduced by increasing the stiffness of the rail, and would depend mainly on the compressibility of the ballast. That was certainly true, but for the investigation dealt with in the Paper, the average depth of the depression in which the wheel moved was not the real consideration; it was the up-and-down movements and the vertical accelerations associated therewith which were under consideration. Those up-and-down movements were greatly reduced by the use of a stiffer rail, and that fact was clearly confirmed by the interesting graphs which Mr. Thompson showed at the Meeting. Both Mr. Gelson and Dr. Lowe-Brown commented on the fact that the values of p stated in the Paper appeared to underestimate the yield of ballast. In answer to that it should be mentioned that ballast did not behave as a perfectly elastic material over its whole range of compression. For a small load the compression was much greater, and there was apparently a considerable amount of initial back-lash to be taken up before the true elasticity of the ballast was revealed. The value $p = 125$, which was stated as corresponding to a ballast of normal rigidity, gave the elasticity of the ballast when the back-lash had been taken up and consolidated by a considerable pressure. It was the figure which actually corresponded to additional deflections given by additional loads, and it was also the figure which accounted for the observed natural frequency for vertical oscillations of a pair of wheels and axle, which in addition to its own weight was supporting a downward force of 10 tons. Probably the mean depths given on some of the figures were understatements, but the movements above and below the mean were the important consideration, and for determining those the values of p given in the Paper appeared to be the most accurate values which could be taken.

Dr. Lowe-Brown put the pertinent question whether or not the improvement obtained by a stiffer rail could not as readily be obtained by increasing the number of sleepers. That was rather a problem for the maintenance engineer, but in a general way it would seem that an increase in the number of sleepers was not likely to effect a reduction in maintenance-costs, and at a rail-joint the sleepers were already as close as was practicable. Decreasing the pitch of sleepers would doubtless give smoother running along continuous rail, but to improve the running over rail-joints a stiffer rail, and the stronger fish-plate which could be introduced thereby, seemed to be the most obvious solution.

THE VERTICAL PATH OF A WHEEL MOVING ALONG A RAILWAY TRACK.

PLATE 1.
THE VERTICAL PATH OF A WHEEL MOVING
ALONG A RAILWAY TRACK.

FIG: 3.

FIGS: 4.

(a)

Wheel-path when sleepers are rigidly supported 0·000901 in.

Wheel-path when ballast-characteristic is $p = 250$ 0·001100 in.

Wheel-path when ballast-characteristic is $p = 125$ 0·001328 in.

Wheel-path when ballast-characteristic is $p = 62.5$ 0·001808 in.

LONG FISHPLATE.

(b)

Wheel-path when sleepers are rigidly supported 0·000465 in.

Wheel-path when ballast-characteristic is $p = 250$ 0·000865 in.

Wheel-path when ballast-characteristic is $p = 125$ 0·001270 in.

Wheel-path when ballast-characteristic is $p = 62.5$ 0·001870 in.

SHORT FISHPLATE.

PATH OF A WHEEL MOVING SLOWLY OVER A RAIL-JOINT.
(MOMENT OF INERTIA OF RAIL-SECTION 36 INCH⁴ UNITS.)

FIGS: 10.

SLEEPERS RIGIDLY SUPPORTED.

BALLAST-CHARACTERISTIC $p = 250$.

BALLAST-CHARACTERISTIC $p = 125$.

BALLAST-CHARACTERISTIC $p = 62.5$.
LONG FISHPLATE.

SLEEPERS RIGIDLY SUPPORTED.

BALLAST-CHARACTERISTIC $p = 250$.

BALLAST-CHARACTERISTIC $p = 125$.

BALLAST-CHARACTERISTIC $p = 62.5$.
SHORT FISHPLATE.

Wheel-loads {Spring-borne load 5·0 tons per rail
Unsprung " 0·9 ton " " "

PATH OF WHEEL CROSSING A RAIL-JOINT AT VARIOUS SPEEDS.
(MOMENT OF INERTIA OF RAIL 36 INCH⁴ UNITS.)
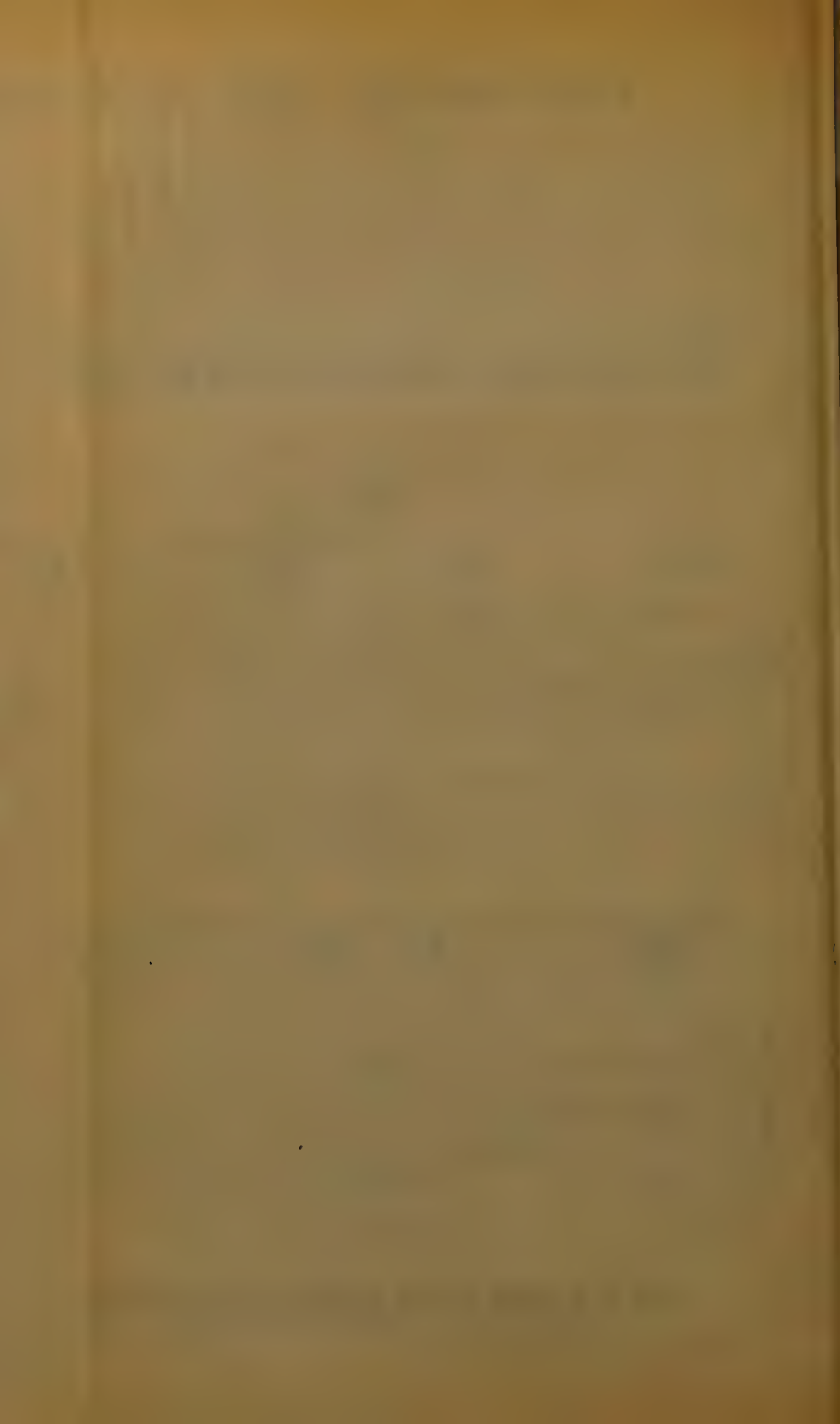
C. E. INGLIS.

SUPPLEMENTARY MEETING.

14 February, 1939.

Professor CHARLES EDWARD INGLIS, O.B.E., M.A., LL.D., F.R.S.,
Vice-President, in the Chair.

The following Paper was submitted for discussion, and, on the
motion of the Chairman, the thanks of The Institution were accorded
to the Authors.

Paper No. 5195.

" On the Problem of Stiffened Suspension-Bridges, and its Treatment by Relaxation Methods." †

By RONALD JOHN ATKINSON, B.E., and Professor RICHARD VYNNE
SOUTHWELL, M.A., F.R.S.

TABLE OF CONTENTS.

INTRODUCTION.

1. THE theory of the stiffened suspension-bridge originated with Rankine
and was developed on the assumption that the suspended truss remains
absolutely rigid under the action of live load ; further development was
made by Professor J. Melan [1], who first took into consideration the deflexion
of the span-truss.   The application of this theory to the design of large
American suspension-bridges, such as the Manhattan bridge, in New York,
and the Philadelphia-Camden bridge [2], showed that this more exact theory
is of great practical importance ; furthermore, it resulted in a considerable
economy of material.

In applying Professor Melan's theory to determine the additional
horizontal component of the cable-stress due to any cause, such as live
load or temperature-change, the assumption has been made that not only

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† Correspondence on this Paper can be accepted until the 15th June, 1939.—
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[1] J. B. Johnson, C. W. Bryan, and F. E. Turneaure, " The Theory and Practice
of Modern Framed Structures," vol. ii, p. 276.   Ninth edition, New York, 1917.

[2] " Quantities of Materials and Costs per Square Foot of Floor for Highway and
Electric-Railway long-span Suspension Bridges " (contribution to Discussion by L. S.
Moisseiff).   Trans. Am. Soc. C.E., vol. 91 (1927), p. 919.

dead load but also additional load may be considered as uniformly dis-
tributed along the span.   In this manner an equation has been established
from which the additional cable-stress can be calculated by a " cut-and-
try " method.   The assumption of uniform load-distribution is sufficiently
accurate provided the span is fully loaded, or nearly so ; but a considerable
error may occur when only a small part of the structure is loaded [1].

2. This account has been taken from a Paper by S. Timoshenko [2],
of which the main concern was with the simplified problem shown in
Fig. 1.   It assumed (in effect) that the number of the vertical suspen-
sion-rods is so great that the loading which they transfer from the girder
to the cable, and hence the curve which is assumed by the latter, may be
treated as continuous.   The assumption is evidently legitimate in a treat-
ment intended to elucidate principles, and it will be made in this Paper.
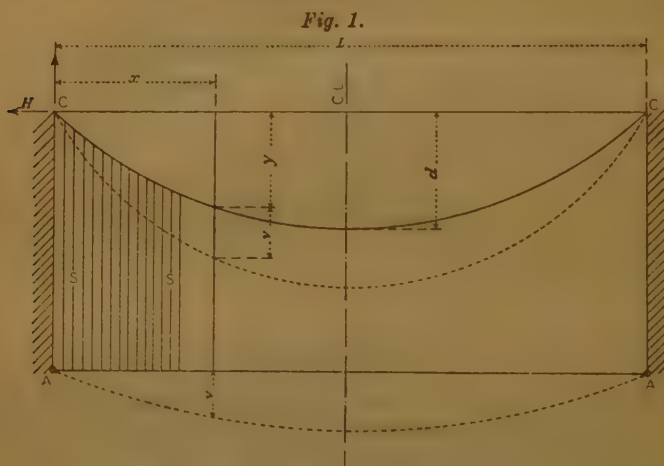
Fig. 1.

Note : Suspension Rods Merely Diagrammatic.

3. Simplified in this way with a view to theoretical treatment, each
" truss " of a suspension-bridge consists (*Fig. 1*) of

    (*a*) an elastic girder AA, with ends constrained in some definite
        manner, to which the load-system is applied ;

    (*b*) vertical suspension-rods S, S, etc., attached to AA, whereby part
        of the load-system is transferred to

    (*c*) a flexible cable CC, deriving its ability to sustain vertical loads
        from tension combined with curvature.

A truss of this kind is " redundant ", whether the ends of the girder be

[1] J. B. Johnson, C. W. Bryan, and F. E. Turneaure, " The Theory and Practice of
Modern Framed Structures," vol. ii, p. 312.   Ninth edition, New York, 1917.
[2] " The Stiffness of Suspension Bridges," Trans. Am. Soc. C.E., vol. 94 (1930),
pp. 377–405.

clamped or simply supported ; for the cable-tension is not statically deter-
minate, but may be altered by self-straining due to temperature-changes
and the like. Usually the lengths of S, S, etc., are so designed that under
the dead loading (w_D) the girder is practically straight and therefore
devoid of stress. The problem now under discussion is to decide what
increase of tension in the cable, what loads in the suspension-rods, and
what bending moments in the girder will be induced by a specified live
loading (w_L).

For simplicity that case is considered in which

- (1) the ends of the girder are simply supported,
- (2) the rods S, S, etc., are inextensible and so long that they remain
    sensibly vertical in the strained configuration,
- (3) the rods are spaced so closely that displacements due to live load
    may be treated as continuous functions of the horizontal
    distance from one end.

In the numerical work it is assumed further (4) that the girder has
uniform flexural rigidity.

Basic Theory.

4. Let x stand for horizontal distances measured from the left-hand
abutment ; let the shape of the cable be expressed by $\bar{y}$, its depth at
x below the level of its end ($x = 0$) ; and let the deflexion of the girder at
x be $\bar{v}$ measured vertically from the horizontal line through its end supports.
Let $\overline{H}$ be the (uniform) horizontal component of the tension in the cable.

Then $- \overline{H}\dfrac{d^2\bar{y}}{dx^2}$ measures the vertical loading at a section defined by x

which the cable sustains in virtue of its tension, and $\dfrac{d^2}{dx^2}\left(B\dfrac{d^2\bar{v}}{dx^2}\right)$ measures
the vertical loading at a section defined by x which the girder sustains in
virtue of its elasticity. Accordingly, *if it may be assumed that the down-
ward pull of the suspension-rod on the cable at x is transmitted as an
upward loading to the girder at x,* then

$$\frac{d^2}{dx^2}\left(B\frac{d^2\bar{v}}{dx^2}\right) - \overline{H}\frac{d^2\bar{y}}{dx^2} = \bar{w},$$

$\bar{w}$ denoting the intensity of the applied loading at a horizontal distance
x from the left-hand abutment. This will be termed the "governing
equation" of the suspension-truss (*Fig. 1*).

The assumption stated in italics was made by Timoshenko in his Paper
(footnote (2), p. 290). He appears to assume in addition (tacitly)
that the displacement of the cable is purely vertical, since no account of
horizontal components is taken in his analysis. On that understanding

(w_D representing the dead loading) initially, at x,

  $\bar{v} = v_0$, the initial deflexion of the girder,
  $\bar{y} = y$, the designed depth of the cable,
  $\bar{H} = H$ (say),

and hence, according to the above equation

$$\frac{d^2}{dx^2}\left(B\frac{d^2v_0}{dx^2}\right) - H\frac{d^2y}{dx^2} = w_D. \quad . \quad . \quad . \quad . \quad (1)$$

When a live load w_L is operative, the conditions at x are

$$\bar{w} = w_D + w_L,$$
$$\bar{v} = v_0 + v \text{ (say)},$$
and $$\bar{y} = y + v,$$

because (according to assumption (2) of section 3) the same additional deflexion must be given both to the cable and to the girder. The tension in the cable will in general be altered, so that

$$\bar{H} = H + H_L \text{ (say)}.$$

Therefore according to the governing equation

$$\frac{d^2}{dx^2}\left\{B\frac{d^2}{dx^2}(v_0 + v)\right\} - (H + H_L)\frac{d^2}{dx^2}(y + v) = w_D + w_L. \quad . \quad (2)$$

By subtraction of (1) from (2) the following is obtained as the governing equation for v

$$\frac{d^2}{dx^2}\left(B\frac{d^2v}{dx^2}\right) - (H + H_L)\frac{d^2v}{dx^2} - H_L\frac{d^2y}{dx^2} = w_L, \quad . \quad . \quad (3)$$

in which on the left-hand side, of the total (specified) live load w_L, the first term gives the part sustained by the girder and the other terms give the part sustained by the cable.

Equation (3) is identical (allowing for changes in notation) with Professor Timoshenko's equation (4) : y being regarded as known (it is under the control of the designer), it can (theoretically) be solved for v as soon as H_L has been determined. This is the essential feature of the problem : it means (since H_L depends on v) that the principle of superposition is not applicable, as in the majority of structural problems.

5. It is at this point that (as the Authors believe) Timoshenko's treatment falls into error by neglecting to consider that sections of the cable will be displaced horizontally as well as vertically. That horizontal displacements must occur is clear when it is realized that otherwise the cable would be stretched, in different parts of its length, by amounts which in general are not consistent with the basic assumption of a uniform horizontal component of tension. In other words, horizontal forces would be required to maintain deflexions restricted to the vertical direction, and on this account Timoshenko's formula for H_L (which he deduces by an argument from strain-energy) is not correct.

To support this contention the Authors have made, in the investigation which follows, allowance for a horizontal displacement u. If u were zero everywhere, then (on the assumption that the cable-tension has a constant horizontal component) the right-hand side of (6) would have to vanish for all values of x, and this requirement is not reconcilable with an unrestricted form for v.

6. In *Fig. 2*, which relates to any cable, the length of the arc between 0 and x is

$$s = \int_0^x \frac{ds}{dx}\, dx = \int_0^x (1 + y'^2)^{\frac{1}{2}} dx,$$

y' denoting the differential dy/dx; and it follows that the change in s ($\varDelta_y s$,

Fig. 2.
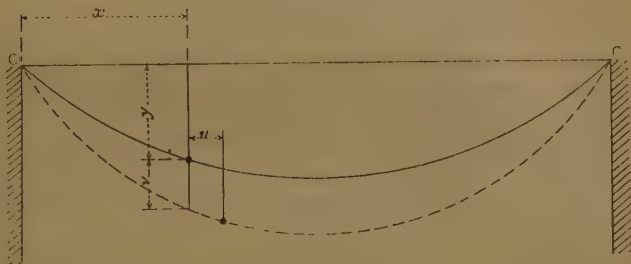
say) due to purely vertical displacements v is given, to the first order in v, by

$$\varDelta_y s = \int_0^x \varDelta_y \left(\frac{ds}{dx}\right) dx = \int_0^x \frac{\frac{dy}{dx} \cdot \varDelta_y \left(\frac{dy}{dx}\right)}{\frac{ds}{dx}}\, dx = \int_0^x \left(\frac{dy}{ds}\right)_x \cdot \frac{dv}{dx}\, dx.$$

If x is altered to $(x + u)$ a further change in s will result, given when u is small by

$$\varDelta_x s = u \frac{d}{dx}(s + \varDelta_y s) = u \frac{ds}{dx} \text{ to the first order of small quantities.}$$

Therefore, if a given section of the cable moves from x on the first curve (y) to a point $(x + u)$ on the second curve $(y + v)$, the total change in the length s up to this section will be given by

$$\varDelta \epsilon = \varDelta_y s + u \frac{ds}{dx} = \int_0^x \left(\frac{dy}{ds}\right)_x v' dx + u s', \quad . \quad . \quad . \quad (4)$$

"dashes" denoting, as before, differentiations with respect to x.

Another expression for $\varDelta \epsilon$ can be deduced from the elastic properties of the cable. For in the first configuration, if T denotes the tension at the

point considered and H its (uniform) horizontal component, the total
extension of the cable up to the section x is

$$\epsilon = \int_0^x \left(\frac{T}{EA}\frac{ds}{dx}\right)dx = \frac{H}{EA}\int_0^x \left(\frac{ds}{dx}\right)^2 dx$$
$$= \frac{H}{EA}\int_0^x (1 + y'^2)dx.$$

Therefore

$\Delta\epsilon = $ total change in ϵ due to u and v combined (and allowing for a
  change ΔH in H)

$$= u\frac{\partial\epsilon}{\partial x} + \Delta_y\epsilon, \text{ in the notation used above,}$$

$$= u\frac{H}{EA}s'^2 + \frac{\Delta H}{EA}\int_0^x s'^2 dx + \frac{2H}{EA}\int_0^x y'v'dx, \quad \ldots \quad (5)$$

to the first order of small quantities.

By elimination of $\Delta\epsilon$ from (4) and (5), there results

$$us'\left(1 - \frac{H}{EA}s'\right) = \frac{\Delta H}{EA}\int_0^x s'^2 dx + \frac{2H}{EA}\int_0^x y'v'dx - \int_0^x \left(\frac{dy}{ds}\right)_x v'dx \quad (6)$$

as an expression from which u can (theoretically) be calculated when v is
specified. For since u must vanish when $x = L$, ΔH can be determined
from the equation

$$0 = \frac{\Delta H}{EA}\int_0^L s'^2 dx + \frac{2H}{EA}\int_0^L y'v'dx - \int_0^L \left(\frac{dy}{ds}\right)_x v'dx, \quad \ldots \quad (7)$$

and on substitution of this expression for ΔH in (6) an equation is
obtained in which u is the only unknown.

7. u having been shown to be significant, it now remains to investigate
the corrections which must be made on its account in the governing
equation (3). First the effects of u and v on the upward loading trans-
mitted to the girder at x will be considered.

In the first configuration of section 6 this loading was given by

$$w_c = - H\frac{d^2y}{dx^2} \quad \ldots \quad \ldots \quad \ldots \quad (8)$$

but in the second configuration the force transmitted to a length δx of the
girder at x is the pull on a horizontal length $\delta x\left(1 + \frac{du}{dx}\right)$ at $(x + u)$ on the
deflected cable : that is to say, the new intensity of upward loading on
the girder at x is

$$w_c + \Delta w_c = \left(1 + \frac{du}{dx}\right)\left(\overline{w} + u\frac{d\overline{w}}{dx}\right)_x,$$

where
$$\overline{w} = - (H + \Delta H)\frac{d^2}{dx^2}(y + v).$$
$\left.\begin{array}{l}\\\\\end{array}\right\} \quad \ldots \quad (9)$

Hence,

$$\Delta w_c = \overline{w} - w_c + \frac{d}{dx}(u\overline{w}),$$ to the first order of small quantities,

$$= -\Delta H \frac{d^2 y}{dx^2} - (H + \Delta H)\left[\frac{d^2 v}{dx^2} + \frac{d}{dx}\left(u \frac{d^2 y}{dx^2}\right)\right],$$ to the same order, by (8) and (9) ;   .   .   (10)

and it follows, if the first and second configurations are identified with those of section 4 (so that H_L can be written for ΔH), that according to the foregoing analysis equation (3) must be replaced by

$$\frac{d^2}{dx^2}\left(B \frac{d^2 v}{dx^2}\right) - (H + H_L)\left[\frac{d^2 v}{dx^2} + \frac{d}{dx}\left(u \frac{d^2 y}{dx^2}\right)\right] - H_L \frac{d^2 y}{dx^2} = w_L \quad . \quad (11)$$

The term involving u is new.

8. $H_L(= \Delta H)$ and u being related with v by (6) and (7), equation (11) determines the vertical deflexion v, common to the cable and to the girder at their sections defined by x (section 4), which results from the imposition of a live loading w_L. Of this loading a part

$$w_G = \frac{d^2}{dx^2}\left(B \frac{d^2 v}{dx^2}\right)$$

is sustained by the girder, and a part

$$w_C = -H_L \frac{d^2 y}{dx^2} - (H + H_L)\left[\frac{d^2 v}{dx^2} + \frac{d}{dx}\left(u \frac{d^2 y}{dx^2}\right)\right]$$

$$\left.\right\} \quad . \quad (12)$$

is sustained by the cable. (w_C, for convenience, here replaces Δw_C in equation (10).)

Now let M_C be defined by the equation

$$-\frac{d^2}{dx^2}M_C = w_C \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

combined with the condition that M_C vanishes at either end of the span. Substituting this expression for w_C in the second of (12), the resulting equation can be integrated to obtain

$$M_C = H_L \cdot y + (H + H_L)\left[v + \int_0^x uy'' dx - \frac{x}{L}\int_0^L uy'' dx\right], \quad (14)$$

in which y'' stands for $d^2 y/dx^2$.

For w_L, in (3), may be substituted its expression in terms of the total moment M_L due to live load. This is defined by the equation

$$-\frac{d^2 M_L}{dx^2} = w_L \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

combined with the condition that M_L vanishes at either end of the span.
In view of the meaning which has been attached to w_C, M_C is that part
of M_L which is transmitted from the girder to the cable.   The remainder
(M_G, say) must be sustained by the girder in virtue of its flexural rigidity,
and on integration the first of (12) gives

$$M_G = M_L - M_C = - B\frac{d^2v}{dx^2}, \quad . \quad . \quad . \quad . \quad (16)$$

since M_G, by analogy with (8), (13), and (15), is the solution of

$$-\frac{d^2}{dx^2}M_G = w_G \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

which vanishes at either end of the span.

9. In (14) and (16) expressions have been obtained for the live-load
moments sustained by the cable and girder, respectively, when a vertical
displacement v (measured from the " dead-load configuration ") is
imposed on both.   The next problem is to determine v when their sum
$M_L(= M_C + M_G)$ is specified [1].

Exact treatment being difficult for the reason that superposition is not
legitimate (section 4), the required solution is sought indirectly (follow-
ing the standard procedure of the relaxation method) by investigating the
forms assumed by M_C and M_G when v has certain assumed distributions.
The first step is to substitute for u in (14).

In (6) and (7) the following may be written,

$$\Delta H = H_L, \quad \frac{dy}{ds} = \frac{dy}{dx}\Big/\frac{ds}{dx} = y'/s', \quad s'^2 = 1 + y'^2 \quad . \quad . \quad (i)$$

Then if the integrals involving v' are modified according to the rules of
" integration by parts " they take the forms

$$us'\left(1 - \frac{H}{EA}s'\right) = \frac{H_L}{EA}\int_0^x s'^2 dx - \frac{vy'}{s'}\left(1 - \frac{2H}{EA}s'\right)$$
$$+ \int_0^x \frac{vy''}{s'^3}\left(1 - \frac{2H}{EA}s'^3\right)dx$$

and

$$0 = \frac{H_L}{EA}\int_0^L s'^2 dx + \int_0^L \frac{vy''}{s'^3}\left(1 - \frac{2H}{EA}s'^3\right)dx,$$

$$\left.\right\} \quad . \quad . \quad (18)$$

since

$$\frac{d}{dx}\left(\frac{y'}{s'}\right) = \frac{y''s' - s''y'}{s'^2} \text{ and } s's'' = y'y'', \text{ according to (i).} \quad . \quad (ii)$$

Eliminating H_L between the two equations (18), u is expressed in terms
of v and known quantities (y', s', etc.).

The relations can be simplified when it is observed that H/EA, being the

fractional extension in the cable when subjected only to dead loading, will in practice be extremely small, whilst s' will not exceed 1·25 (say). In consequence all terms which involve H/EA may be neglected in (18), and on that understanding these equations reduce to

$$\left. \begin{array}{l} \dfrac{H_L}{EA}\displaystyle\int_0^L s'^2 dx = -\int_0^L \dfrac{vy''}{s'^3} dx, \\[3mm] us' = -\dfrac{vy'}{s'} + \displaystyle\int_0^x \dfrac{vy''}{s'^3}\, dx + \dfrac{H_L}{EA}\int_0^x s'^2 dx. \end{array} \right\} \quad . \quad . \quad (19)$$

It can be seen from (18) and (19) that, provided y is symmetrical with respect to the central section (as is usually true in practice), no contribution is made to H_L by any deflexion v which is distributed skew-symmetrically with respect to the centre. Further progress is impossible until some definite form has been assumed for y.

10. In equation (1), w_D being specified, the distribution either of v_0 or of y is at the choice of the designer (subject to the condition that both quantities must vanish at each end), because (section 3) the structure is redundant and as such can be self-strained. Since $w_D = -d^2 M_D/dx^2$, where M_D is the bending moment corresponding with the dead load, a double integration of (1) yields the equivalent relation

$$- B\frac{d^2 v_o}{dx^2} + Hy = M_D, \quad . \quad . \quad . \quad . \quad . \quad \text{(iii)}$$

which shows (in accordance with the theory of the " funicular ") that v_o will vanish everywhere, leaving the girder free from stress, if the shape and tension of the cable are so chosen that

$$Hy = M_D \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(iv)}$$

(for example, by making y parabolic when w_D is uniform). Otherwise some deflexion must be given to the girder, since $(Hy - M_D)$ cannot then vanish everywhere.

Usually y is determined without special reference to the form of M_D, and in that event either of two alternatives may be adopted: either v_0 may be deduced from (iii) or, v_0 being kept zero, a value of H may be chosen such that Hy is approximately equal to M_D everywhere, and the remainder of M_D then treated as an addition to the " live-load moment " M_L. It will be assumed that the second alternative is chosen, also that y is given by the expression

$$y = \frac{1}{k}\left\{\cosh \frac{kL}{2} - \cosh k\left(x - \frac{L}{2}\right)\right\}, \quad . \quad . \quad . \quad . \quad (20)$$

which defines the " uniform catenary." By putting $x = \dfrac{L}{2}$ it is found that

the dip/span ratio

$$\frac{d}{L} = \frac{1}{kL}\left(\cosh\frac{kL}{2} - 1\right) = \frac{2}{kL}\sinh^2\frac{kL}{4}, \quad . \quad . \quad . \quad (21)$$

so that kL will be small when d/L is small. Then to a close approximation

$$\frac{d}{L} \approx \frac{kL}{8}, \qquad y = d\left\{1 - \frac{\sinh^2\frac{1}{2}k\left(x - \frac{L}{2}\right)}{\sinh^2\frac{kL}{4}}\right\}$$

$$\approx d\left\{1 - \left(2\frac{x}{L} - 1\right)^2\right\} = 4d\frac{x}{L}\left(1 - \frac{x}{L}\right), \quad (22)$$

so the shape of the cable is approximately parabolic. This is in accord with standard practice, and (22) might be taken as exact relations; but analytically the assumption (20) is more convenient, and it will introduce no difficulty in design.

11. According to (20),

$$\left.\begin{array}{l} y' = -\sinh k\left(x - \frac{L}{2}\right), \;\; y'' = -k\cosh k\left(x - \frac{L}{2}\right), \\[2mm] s' = \sqrt{(1 + y'^2)} = \cosh k\left(x - \frac{L}{2}\right), \end{array}\right\} \quad . \quad . \quad (23)$$

and on this understanding equations (19) become

$$\left.\begin{array}{l} \dfrac{1}{2}L\left(1 + \dfrac{\sinh kL}{kL}\right)\dfrac{H_L}{EA} = k\displaystyle\int_0^L v\,\mathrm{sech}^2\,k\left(x - \dfrac{L}{2}\right)dx, \\[3mm] u\cosh k\left(x - \dfrac{L}{2}\right) = \dfrac{1}{4k}\{2kx + \sinh kL + \sinh k(2x - L)\}\dfrac{H_L}{EA} + \\[3mm] \qquad + v\tanh k\left(x - \dfrac{L}{2}\right) - k\displaystyle\int_0^x v\,\mathrm{sech}^2\,k\left(x - \dfrac{L}{2}\right)dx. \end{array}\right\} \quad (24)$$

Also, (14) becomes

$$M_C = H_L \cdot y + (H + H_L)\left[v - k\int_0^x u\cosh k\left(x - \frac{L}{x}\right)dx \right.$$
$$\left. + \frac{kx}{L}\int_0^L u\cosh k\left(x - \frac{L}{2}\right)dx\right], \quad . \quad (25)$$

in which y has the expression (20). Thus the assumption made in regard to the form of the cable makes it easy to eliminate u between (25) and the second of (24). The integrals are, however, intractable in their present form.

Now the Fourier expansion

$$v \operatorname{sech}^2 k\left(x - \frac{L}{2}\right) = V_1 \sin (\pi x/L) + V_2 \sin (2\pi x/L) + \ .\ . \quad (26)$$

is admissible, since v (in the nature of the case) vanishes at both ends of the span and is everywhere finite and continuous. It may be integrated term by term, and both sides of (26) may be multiplied by any finite quantity. With this substitution, equation (24) can be replaced by

$$\frac{1}{2}\left(1 + \frac{\sinh kL}{kL}\right)\frac{H_L}{EA} = \frac{k}{\pi}\Sigma_n[V_n(1 - \cos n\pi)/n] \quad . \quad . \quad (27)$$

and

$$u \cosh k\left(x - \frac{L}{2}\right) = \frac{1}{4k}\{2kx + \sinh kL + \sinh k(2x - L)\}\frac{H_L}{EA} +$$

$$+ \frac{1}{2}\sinh k(2x - L)\Sigma_n[V_n \sin (n\pi x/L)] - \frac{kL}{\pi}\Sigma_n\left[\frac{V_n}{n}\{1 - \cos (n\pi x/L)\}\right],$$

and on integrating the second of these equations it is found that the expression (25) for M_C can be replaced by

$$M_C = H_L \cdot y + (H + H_L)\left[\cosh^2 k(x - L/2)\Sigma_n\left[V_n \sin (n\pi x/L)\right]\right.$$

$$+ \frac{1}{4}\left\{kx(L - x) + \frac{\cosh kL - \cosh k(2x - L)}{2k}\right\}\frac{H_L}{EA}$$

$$- \frac{k^2L^2}{\pi^2}\cosh k(2x - L)\Sigma_n\left[\frac{V_n \sin (n\pi x/L)}{n^2 + 4k^2L^2/\pi^2}\right]$$

$$+ \frac{kL}{2\pi}\sinh k(2x - L)\Sigma_n\left[\frac{nV_n \cos (n\pi x/L)}{n^2 + 4k^2L^2/\pi^2}\right]$$

$$+ \frac{kL}{2\pi}\sinh kL\ \Sigma_n\left[\frac{nV_n}{n^2 + 4k^2L^2/\pi^2}\right]$$

$$- \frac{k^2L^2}{\pi^2}\Sigma_n\left[\frac{V_n}{n^2}\sin (n\pi x/L)\right]$$

$$\left.\left.- \frac{kx}{2\pi}\sinh kL\ \Sigma_n\left[\frac{nV_n(1 + \cos n\pi)}{n^2 + 4k^2L^2/\pi^2}\right]\right]\right], \quad . \quad . \quad . \quad . \quad . \quad (28)$$

H_L being given its value according to (27). It is easy to verify that this expression vanishes at both ends of the span.

12. It is now possible to assess the practical significance of the horizontal displacements which (in sections 5 and 6) have been shown to be essential to any exact treatment. In (28), the first line represents the terms

$$H_L \cdot y + (H + H_L)v$$

which would be obtained according to Timoshenko's equation (3); the
remaining terms appear as a consequence of the displacements u.

The co-factor of H_L/EA, in the second line, may be replaced by

$$\frac{1}{4}\left[kx(L - x) - \frac{1}{k} \sinh\,kx\,\sinh\,k(L - x) \right],$$

and the expression (20) for y may be replaced in the same way by

$$y = -\frac{2}{k}\sinh\frac{kx}{2}\sinh\frac{k}{2}(L - x).$$

Therefore the contribution to M_C of the second line in (28) may be written
in the form

$$H_L \cdot y \times \frac{H + H_L}{EA}\left(\frac{-\alpha\beta}{2\sinh\alpha\sinh\beta} + \frac{1}{2}\cosh\alpha\cosh\beta \right) \quad . \quad \text{(v)}$$

where (for brevity) α and β replace $\dfrac{kx}{2}$ and $\dfrac{k(L - x)}{2}$ respectively. Now
in practice $kL/2$ ($\approx 4d/L$ according to section 10), and therefore α and β,
will not exceed 0·5: therefore, in view of its factor $(H + H_L)/EA$, the
quantity (v) can always be neglected in comparison with the term $H_L \cdot y$
which appears in the first line of (28).

13. The remainder of the terms due to u are most conveniently studied
by calculating the contributions which they make to the various harmonic
components of M_C. Since M_C vanishes at both ends of the span, it can
be represented by the Fourier series

$$M_C = (M_C)_1 \sin(\pi x/L) + (M_C)_2 \sin(2\pi x/L) + \ldots \text{etc.} \quad . \quad \text{(29)}$$

where

$$(M_C)_\lambda = \frac{2}{L}\int_0^L M_C \sin\lambda\pi\frac{x}{L}dx. \quad . \quad . \quad . \quad . \quad . \quad \text{(30)}$$

Dealing with the remaining terms in this way, it is found that their
contribution to $(M_C)_\lambda$ is given by $(H + H_L)$ multiplied by

$$-\frac{k^2L^2}{\lambda^2\pi^2}V_\lambda + \frac{kL\sinh\,kL}{\lambda\pi^2}$$

$$\times \Sigma_n\left[\frac{n\{1 + \cos(\lambda - n)\pi\}V_n(n^2 - \lambda^2 + 4k^2L^2/\pi^2)}{(\lambda^2 - n^2)^2 + 8(\lambda^2 + n^2)k^2L^2/\pi^2 + 16k^4L^4/\pi^4} \right], \quad . \quad \text{(31)}$$

vanishing when λ differs from n by an odd integer. This may be compared
with the contribution to $(M_C)_\lambda$ of the term $(H + H_L)v$ in the first line

of (28)—namely $(H + H_L)$ multiplied by

$$\frac{1}{2}V_\lambda + \frac{4\lambda}{\pi^2}kL \sinh kL$$

$$\times \Sigma_n \left[\frac{n\{1 + \cos(\lambda - n)\pi\}V_n}{(\lambda^2 - n^2)^2 + 8(\lambda^2 + n^2)k^2L^2/\pi^2 + 16k^4L^4/\pi^4} \right] \quad . \quad . \quad (32)$$

The contributions of V_n according to (31) and (32) are in the ratio

$$\left. \begin{aligned} -\frac{k^2L^2}{\lambda^2\pi^2} \cdot \frac{2 - \dfrac{\sinh kL}{kL} + 2\dfrac{k^2L^2}{\lambda^2\pi^2}}{1 + \dfrac{\sinh kL}{kL} + \dfrac{k^2L^2}{\lambda^2\pi^2}}, \quad \text{when } n = \lambda, \\[2ex] \text{and in the ratio} \\[1ex] \frac{1}{4\lambda^2}\left(n^2 - \lambda^2 + \frac{4k^2\lambda^2}{\pi^2}\right), \quad \text{when } n \neq \lambda. \end{aligned} \right\} \quad . \quad . \quad (33)$$

Thus when $n = \lambda$ they are roughly in the ratio $-\dfrac{1}{2}\dfrac{k^2L^2}{\lambda^2\pi^2}$, which will be small in practice—more especially as regards the higher harmonics, since kL (see section 12) will not exceed 1. When $n \neq \lambda$ they are of the same order ; but both will be zero when $(\lambda - n)$ is an odd integer, and both will be small (of the order $\dfrac{2\dfrac{n}{\lambda}k^2L^2}{(n^2 - \lambda^2)\pi^2}$) when $(\lambda - n)$ is an even integer. Thus it is only in relation to the first few terms of (29) that the effects of u are significant.

14. A further contribution to $(M_C)_\lambda$ in the series (29) comes from the term $H_L \cdot y$ in (28). According to (30), when y is given by (20) this is

$$\frac{2H_L}{kL}\int_0^L \left\{ \cosh \frac{kL}{2} - \cosh k\left(x - \frac{L}{2}\right) \right\} \sin \lambda\pi\frac{x}{L}dx$$

$$= (1 - \cos \lambda\pi)\frac{2kL \cosh \dfrac{kL}{2}}{\lambda^3\pi^3\left(1 + \dfrac{k^2L^2}{\lambda^2\pi^2}\right)}LH_L \quad . \quad . \quad (34)$$

Equations (31), (32) and (34) give a complete expression for $(M_C)_\lambda$; so now the series (29) is definite and can be used in place of (28).

15. Corresponding with (29) the Fourier series

$$M_G = (M_G)_1 \sin(\pi x/L) + (M_G)_2 \sin(2\pi x/L) + \ldots \quad . \quad . \quad (35)$$

may be used as a valid expression for M_G ; and a formula similar to (30),

applied to (16), gives

$$(M_G)_\lambda = -\frac{2B}{L}\int_0^L \frac{d^2v}{dx^2}\sin \lambda\pi \frac{x}{L}\,dx \quad (B \text{ being uniform}),$$

$$= \lambda^2\pi^2\frac{B}{L^2}\times \frac{2}{L}\int_0^L v\sin\lambda\pi\frac{x}{L}\,dx, \text{ on an integration by parts .} \quad (36)$$

Referring to section 13 it can be seen that the co-factor of $\lambda^2\pi^2 B/L^2$, in this expression, is the quantity given in (32). Therefore if

$$M_L = M_G + M_C \quad . \quad . \quad . \quad . \quad . \quad (16)\;bis$$

be similarly expressed by a Fourier series

$$\left.\begin{array}{l} M_L = (M_L)_1\sin (\pi x/L) + (M_L)_2\sin (2\pi x/L) + \ldots \\[1.2em] \text{in which by (16), (29), and (35)} \\[0.6em] (M_L)_\lambda = (M_G)_\lambda + (M_C)_\lambda, \end{array}\right\} \quad (37)$$

then in the expression for $(M_L)_\lambda$ the quantity (32) will appear conjoined with a multiplying factor

$$\lambda^2\pi^2\frac{B}{L^2} + (H + H_L), \quad . \quad . \quad . \quad . \quad (38)$$

of which the first and second terms represent the actions, respectively, of the girder and of the cable. The remainder of $(M_L)_\lambda$ is contributed solely by the cable : it consists of a part (34), due to the term $H_L \cdot y$ in (28), and of a part, given by (31) multiplied by $(H + H_L)$, due to the terms in (28) which come from the terms in u. All but this last part of $(M_L)_\lambda$ is given by Timoshenko's theory (sections 4 and 5).

16. The formulas just obtained involve three parameters (namely, kL, EA, and B/HL^2) which are at the choice of the designer. Before they can be applied to any actual example, values must be assigned to these quantities.

It has been shown (section 10) that kL is approximately equal to $8d/L$. In existing bridges d/L appears to lie between 0·06 and 0·1, and the higher figure corresponds with a value 0·79 of kL : for this value the formulas (31) and (32) of section 13 become

$$-\frac{0\cdot063235}{\lambda^2}V_\lambda +$$

$$+ \frac{0\cdot070021}{\lambda}\Sigma_n\left[\frac{n\{1 + \cos (\lambda - n)\pi\}V_n(n^2 - \lambda^2 + 0\cdot25294)}{(\lambda^2 - n^2)^2 + 0\cdot50588(\lambda^2 + n^2) + 0\cdot063979}\right]$$

$$\text{and} \qquad = \Sigma_n[A_{\lambda,n}V_n]\ (\text{say}) \quad . \quad . \quad . \quad . \quad . \quad (31\text{A})$$

$$\frac{1}{2}V_\lambda + 0\cdot280086\lambda\Sigma_n\left[\frac{n\{1 + \cos (\lambda - n)\pi\}V_n}{(\lambda^2 - n^2)^2 + 0\cdot50588(\lambda^2 + n^2) + 0\cdot063979}\right]$$

$$= \Sigma_n[B_{\lambda,n}V_n]\ (\text{say}), \quad . \quad . \quad . \quad . \quad . \quad (32\text{A})$$

and the expression (34) of section 14, after substitution for H_L from (27), becomes

$$(1 - \cos \lambda\pi)\frac{0.013123EA}{\lambda(\lambda^2 + 0.063235)}\Sigma_n\left[\frac{V_n}{n}(1 - \cos n\pi)\right]$$

$$= \Sigma_n[C_{\lambda,n}V_n] \text{ (say)}. \quad . \quad . \quad . \quad . \quad . \quad (34\text{A})$$

Tables I, II, and III give, respectively, numerical values of $A_{\lambda,n}$, $B_{\lambda,n}$, $C_{\lambda,n}$ for values of λ and of n ranging from 1 to 9.

TABLE I.—VALUES OF $A\lambda,n \times 10^6$, IN EQUATION (31A), WHEN $kL = 0.79$.

Values of n	Values of λ								
	1	2	3	4	5	6	7	8	9
1	−30307	0	− 5232	0	−1129	0	− 410	0	− 193
2	0	− 7193	0	−5335	0	−1419	0	− 576	0
3	50161	0	− 3163	0	−4842	0	−1464	0	− 640
4	0	22259	0	−1772	0	−4324	0	−1426	0
5	28822	0	13882	0	−1132	0	−3872	0	−1360
6	0	12975	0	9978	0	− 786	0	−3492	0
7	20307	0	8072	0	7751	0	− 577	0	−3173
8	0	9287	0	5764	0	6321	0	− 442	0
9	15703	0	5805	0	4446	0	5329	0	− 349

TABLE II.—VALUES OF $B\lambda,n \times 10^6$, IN EQUATION (32A), WHEN $kL = 0.79$.

Values of n	Values of λ								
	1	2	3	4	5	6	7	8	9
1	1020732	0	24312	0	4754	0	1683	0	783
2	0	1045044	0	29066	0	6437	0	2466	0
3	24312	0	1049797	0	30749	0	7220	0	2892
4	0	29066	0	1051480	0	31532	0	7646	0
5	4754	0	30749	0	1052263	0	31958	0	7903
6	0	6437	0	31532	0	1052689	0	32215	0
7	1683	0	7220	0	31958	0	1052946	0	32382
8	0	2466	0	7646	0	32215	0	1053113	0
9	783	0	2892	0	7903	0	32382	0	1053228

TABLE III.—VALUES OF $10^6 \times \dfrac{C_{\lambda,n}}{EA}$, IN EQUATION (34A), WHEN $kL = 0.79$.

Values of n	Values of λ				
	1	3	5	7	9
1	49370·1	1930·58	418·876	152·841	71·9493
3	16465·7	643·528	139·626	50·9469	23·9831
5	9874·02	386·117	83·7753	30·5681	14·3899
7	7052·87	275·797	59·8394	21·8344	10·2785
9	5485·56	214·509	46·5418	16·9823	7·99437

NOTE.—$C_{\lambda,n} = 0$ when either λ or n is even.

NUMERICAL SOLUTIONS BY THE METHOD OF RELAXATION.

17. Collecting the results of sections 11–16,

$$(M_L)_\lambda = (H + H_L)\Sigma_n[A_{\lambda,n}V_n]$$
$$+ (\lambda^2\pi^2\frac{B}{L^2} + H + H_L)\Sigma_n[B_{\lambda,n}V_n] + \Sigma_n[C_{\lambda,n}V_n], \quad . \quad . \quad (39)$$

where according to (27)

$$H_L = \frac{\frac{2}{\pi}EA\,\Sigma_n\left[\dfrac{kV_n(1 - \cos n\pi)}{n}\right]}{1 + \dfrac{\sinh kL}{kL}},$$

$$= \Sigma_n[D_nV_n], \text{ say.} \quad . \quad . \quad . \quad . \quad . \quad . \quad (40)$$

Equations of the same form would be obtained from Timoshenko's equation (3) and his formula corresponding with (but different from) the Authors' formula (7), but the constants $A_{\lambda,n}$, . . . etc. would have values different

TABLE IV.—VALUES OF $D_n \times L/EA \times 10^6$, IN EQUATION (40), WHEN $kL = 0.79$.

	Values of n				
	1	3	5	7	9
$D_n \times \dfrac{L}{EA} \times 10^6$ . .	477317	159106	95463	68188	53035

NOTE.—When n is even or zero, $D_n = 0$.

from those which the Authors have calculated on the basis of their modified analysis. Whatever values be adopted, the essential feature of the problem is the fact that $(M_L)_\lambda$ is not a linear function of the V's, because these also affect the value of H_L. Table IV gives, in the case

where $kL = 0.79$ values of $10^6 \times D_n L/EA$ for values of n ranging from 1 to 9.

For the reason stated, the normal procedure of the relaxation method will require some modification. The most convenient procedure would seem to be, using the equivalent equation

$$(M_L)_\lambda - H_L \Sigma_n[(A_{\lambda,\,n} + B_{\lambda,\,n})V_n]$$

$$= H\Sigma_n[A_{\lambda,\,n} \cdot V_n] + (\lambda^2 \pi^2 \frac{B}{L^2} + H)\Sigma_n[B_{\lambda,\,n}V_n]$$

$$+ \Sigma_n[C_{\lambda,\,n}V_n],$$

$$= H\Sigma_n[F_{\lambda,\,n} \cdot V_n], \text{ say, } \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (41)$$

to start by neglecting the term in H_L on the left-hand side and (by relaxation processes of the normal type) liquidate the $(M_L)_\lambda$'s fairly completely; then to calculate and add to the unliquidated remainders the correcting term $- H_L\Sigma_n[(A_{\lambda,\,n} + B_{\lambda,\,n})V_n]$; then to liquidate the residuals thus corrected; then to correct again; and so on. This means that in the relaxation process (conducted in the usual way) it will be necessary to keep account of H_L as well as of the $(M_L)_\lambda$'s.

In the standard notation of the relaxation method, and omitting the term involving H_L in $(M_L)_\lambda$, the " residual"

$$\left.\begin{array}{c}(\bar{\bar{M}}_L)_\lambda = (M_L)_\lambda - H \cdot \Sigma_n[F_{\lambda,\,n}V_n], \\[2mm] \text{so that initially} \\[1mm] (\bar{\bar{M}}_L)_\lambda = (\bar{M}_L)_\lambda \text{ (given)}, \\[2mm] \text{and in the liquidation process} \\[1mm] \dfrac{(\partial \bar{\bar{M}}_L)_\lambda}{\partial V_n} = -H \cdot F_{\lambda,\,n.}\end{array}\right\} \quad . \quad . \quad . \quad (42)$$

($F_{\lambda,\,n}$ is thus seen to be an " influence coefficient.") The Operations Table must give the effect on $(\bar{\bar{M}}_L)_\lambda$ of all V_n's, also their effect on H_L according to (40).

18. For numerical illustration of the relaxation procedure the following example, which is generally representative of the Manhattan suspension bridge (central span)[1], will be treated.

In *Fig. 1* (p. 290),

$$\left.\begin{array}{l}d = 145 \text{ feet}; \ L = 1{,}450 \text{ feet } (d/L = 0.1; \ kL = 0.79). \\ \text{Also} \\ H = 10.5 \times 10^6 \text{ lb. } (w_D = 5{,}793 \text{ lb. per foot}); \\ A = 275 \text{ square inches}; \\ E = 30 \times 10^6 \text{ lb. per square inch (assumed)}; \\ \quad \text{therefore } EA = 8{,}250 \times 10^6 \text{ lb.; } \ EA/H = 786. \\ B = 1{,}317 \times 10^9 \text{ lb.-foot}^2 \text{ units : therefore } B/HL^2 = 0.0597.\end{array}\right\} \quad . \quad (43)$$

[1] J. B. Johnson, C. W. Bryan, and F. E. Turneaure, " The Theory and Practice of Modern Framed Structures," vol. ii, pp. 241–250. New York, 1929.

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According to (41)

$$M_\lambda - H_L \Sigma_n[(A_{\lambda,n} + B_{\lambda,n})V_n] = \frac{H}{10^6}\Sigma_n[10^6 F_{\lambda,n} . V_n],$$

where

$$10^6 F_{\lambda,n} = 10^6\left[A_{\lambda,n} + B_{\lambda,n} + \frac{\pi^2 B}{HL^2}\lambda^2 B_{\lambda,n} + \frac{EA}{H} \times \frac{C_{\lambda,n}}{EA}\right].$$

Therefore in this example, according to (43), when $H_L = 0$,

$$M_\lambda = 10{\cdot}5\Sigma_n[10^6 F_{\lambda,n}V_n],$$

where

$$\left.10^6 F_{\lambda,n} = 10^6\left[A_{\lambda,n} + B_{\lambda,n} + 0{\cdot}589\lambda^2 B_{\lambda,n} + 786\frac{C_{\lambda,n}}{EA}\right].\right\} \quad . \quad (44)$$

The quantities $10^6 \times \left(A_{\lambda,n}, B_{\lambda,n}, \dfrac{C_{\lambda,n}}{EA}\right)$ are given in Tables I–III. They have been used to calculate values of $10^6 \times F_{\lambda,n}$ as given in Table V, also values of $10^6 \times (A_{\lambda,n} + B_{\lambda,n})$ as given in Table VI, for insertion on the right and left-hand sides, respectively, of equation (41).

For use in conjunction with (42) to investigate different loadings on this particular truss, values of the quantities $(-H . F_{\lambda,n}$ and $D_n)$ have been derived from the foregoing Tables (as distinct from D_nL/EA in Table III). These quantities are recorded in Table VII: $-H . F_{\lambda,n}$ is the influence coefficient measuring (when $H_L . v$ effects are neglected) the change due to V_n in the "residual moment" $(M_L)_\lambda$; D_n measures the effect of V_n on H_L (which will require to be calculated in order that the neglected terms of (41) may be taken into account). Thus Table VII is an "Operations Table" of the standard kind.

19. The relaxation process can now be started, once M_L has been expressed as a Fourier series (37). To illustrate it any specified distribution M_L might have been taken; but results of more general application are obtained by dealing separately with successive terms in (37)— that is, by assuming successively that

$$\left.\begin{array}{l} M_L \propto \sin \pi x/L \\ M_L \propto \sin 2\pi x/L \end{array}\right\} \quad . \quad . \quad . \quad . \quad (45)$$

. . . and so on. A description will now be given of the relaxation process as applied to determine the displacement corresponding with a loading such that $M_L = 10^9 \sin \pi x/L$.

It may be expected that V_1 will predominate in the response to a loading of this nature, and Table VII shows that it has a predominating effect on $(M_L)_1$ and on H_L. Accordingly, for a first approximation all terms in (41) which do not involve V_1 may be neglected: then

$$\begin{array}{ll} H_L = D_1 V_1, & \text{by (40),} \\ \quad = 2715769 V_1, & \text{from Table VII,} \end{array}$$

Table V.—Values of $F\lambda_{,n} \times 10^6$, in Equation (41), when $kL = 0.79$.

Values of n	Values of λ									Sum (required for checking)
	1	2	3	4	5	6	7	8	9	
1	40382216	0	1664796	0	402719	0	169918	0	94464	42714115
2	0	3499096	0	297551	0	141459	0	94815	0	4032921
3	13026124	0	7115251	0	588231	0	254088	0	159021	21142714
4	0	119780	0	10955318	0	695574	0	294341	0	12065013
5	7794534	0	510951	0	16605996	0	974114	0	394759	26280354
6	0	34572	0	338562	0	23365153	0	1242666	0	24980952
7	5564522	0	270249	0	557139	0	31447751	0	1581646	39421308
8	0	17561	0	85440	0	721379	0	40736648	0	41561028
9	4327030	0	192565	0	165248	0	985298	0	51289678	56959818

Table VI.—Values of $(A\lambda_{,n} + B\lambda_{,n}) \times 10^6$, in Equation (41), when $kL = 0.79$.

Values of n	Values of λ									Sum (required for checking)
	1	2	3	4	5	6	7	8	9	
1	990425	0	19080	0	3625	0	1273	0	590	1014993
2	0	1037851	0	23731	0	5018	0	1890	0	1068490
3	74473	0	1046634	0	25907	0	5756	0	2252	1155022
4	0	51325	0	1049708	0	27208	0	6220	0	1134461
5	33576	0	44631	0	1051131	0	28086	0	6543	1163967
6	0	19412	0	41510	0	1051903	0	28723	0	1141548
7	21990	0	15292	0	39709	0	1052369	0	29209	1158569
8	0	11753	0	13410	0	38536	0	1052671	0	1116370
9	16486	0	8697	0	12349	0	37711	0	1052879	1128122

Table VII.—Values of $-HF\lambda_{,n}$ (H in lb. weight) and of D_n (lb.-foot units).

Number and nature of operation	Values of λ									Sum (required for checking)	Values of D_n
	1	2	3	4	5	6	7	8	9		
(1) $V_1 = 1$	−424013250	0	−17480360	0	−4228554	0	−1784140	0	−991876	−448498180	2715769
(2) $V_2 = 1$	0	−36740504	0	−3124287	0	−1485323	0	−995559	0	−42345673	0
(3) $V_3 = 1$	−136774296	0	−74710127	0	−6176421	0	−2667921	0	−1669721	−221998486	905258
(4) $V_4 = 1$	0	−1257691	0	−115030836	0	−7303525	0	−3090577	0	−126682629	0
(5) $V_5 = 1$	−81842604	0	−5364984	0	−174362950	0	−10228201	0	−4144969	−275943708	543152
(6) $V_6 = 1$	0	−363008	0	−3554896	0	−245334090	0	−13047994	0	−262299988	0
(7) $V_7 = 1$	−58427478	0	−2837616	0	−5849962	0	−330201370	0	−16607285	−413923711	387966
(8) $V_8 = 1$	0	−184389	0	−897122	0	−7574480	0	−427734778	0	−436390769	0
(9) $V_9 = 1$	−45433814	0	−2021931	0	−1735101	0	−10345624	0	−538541589	−598078059	301751

Line number.	Operation and multiplier.	Total amplitudes of V components.	$(M_L)_1$	$(M_L)_2$	$(M_L)_3$	$(M_L)_4$	$(M_L)_5$	$(M_L)_6$	$(M_L)_7$	$(M_L)_8$	$(M_L)_9$	Check.	$\Delta H_L \times 10^{-6}$
	Initial $M_L = 10^6 \times$		1000·000	0	0	0	0	0	0	0	0	1000·000	
(1)	$V_1 = 2·415$	$V_1 = 2·415$	−1023·992	0	−42·215	0	−10·212	0	−4·309	0	−2·395	−1083·123	6·5586
			− 23·992	0	−42·215	0	−10·212	0	−4·309	0	−2·395	83·123	
(2)	$V_3 = − 0·564$	$V_2 = − 0·564$	77·141	0	42·137	0	3·484	0	1·505	0	0·942	125·207	−0·5106
			53·149	0	− 0·078	0	− 6·728	0	−2·804	0	−1·453	42·084	
(3)	$V_5 = − 0·0386$	$V_5 = − 0·0386$	3·159	0	0·207	0	6·730	0	0·395	0	0·160	10·651	−0·0210
			56·308	0	0·129	0	0·002	0	−2·409	0	−1·293	52·735	
(4)	$V_7 = − 0·0073$	$V_7 = − 0·0073$	0·427	0	0·021	0	0·043	0	2·410	0.	0·121	3·022	÷0·0028
			56·735	0	0·150	0	0·045	0	0·001	0	−1·172	55·757	
(5)	$V_9 = − 0·00218$	$V_9 = − 0·00218$	0·099	0	0·004	0	0·004	0	0·023	0	1·174	1·304	−0·0007
			56·834	0	0·154	0	0·049	0	0·024	0	0·002	57·061	
(6)	$V_1 = 0·0979$	$V_1 = 2·5129$	− 41·511	0	− 1·711	0	− 0·414	0	−0·175	0	−0·097	− 43·908	0·2659
			15·323	0	− 1·557	0	− 0·365	0	−0·151	0	−0·095	13·153	$H_L = 6·2894 \times 10^6$
(7)	$H_L.v$ effect of V_1 (Table VI)		− 15·653	0	− 0·302	0	− 0·057	0	−0·020	0	−0·009	− 16·042	
(8)	,, ,, V_3		0·264	0	3·713	0	0·092	0	0·020	0	0·008	4·097	
(9)	,, ,, V_5		0·008	0	0·011	0	0·255	0	0·007	0	0·002	0·283	
(10)	,, ,, V_7		0·001	0	0·001	0	0·002	0	0·048	0	0·001	0·053	
(11)	,, ,, V_9		0	0	0	0	0	0	0·001	0	0·014	0·015	
(12)	Residual $M_L = 10^6 \times$		− 0·057	0	1·866	0	− 0·073	0	−0·095	0	−0·079	1·559	$H_L = 6·2894 \times 10^6$
(13)	$V_3 = 0·0253$	$V_3 = − 0·5387$	− 3·460	0	− 1·890	0	− 0·156	0	−0·067	0	−0·042	− 5·617	0·0229
			− 3·517	0	− 0·024	0	− 0·229	0	−0·162	0	−0·121	− 4·058	
(14)	$V_5 = − 0·00131$	$V_5 = − 0·03991$	0·107	0	0·007	0	0·228	0	0·013	0	0·005	0·361	−0·0007
			− 3·410	0	− 0·017	0	− 0·001	0	−0·149	0	−0·116	− 3·697	
(15)	$V_7 = − 0·00045$	$V_7 = − 0·00775$	0·026	0	0·001	0	0·003	0	0·149	0	0·007	0·186	−0·0002
			− 3·384	0	− 0·016	0	0·002	0	0	0	−0·109	− 3·511	
(16)	$V_9 = − 0·00020$	$V_9 = − 0·00238$	0·009	0	0	0	0	0	0·002	0	0·108	0·120	−0·0001
			− 3·375	0	− 0·016	0	0·002	0	0·002	0	−0·001	− 3·391	
(17)	$V_1 = − 0·00795$	$V_1 = 2·50495$	3·371	0	0·139	0	0·034	0	0·014	0	0·008	3·566	−0·0216
			− 0·004	0	0·123	0	0·036	0	0·016	0	0·007	0·175	
(18)	$V_3 = 0·00173$	$V_3 = − 0·53697$	− 0·237	0	− 0·129	0	− 0·011	0	−0·005	0	−0·003	− 0·384	0·0016
			− 0·241	0	− 0·006	0	0·025	0	0·011	0	0·004	− 0·209	
(19)	$V_5 = 0·00014$	$V_5 = − 0·03977$	− 0·011	0	− 0·001	0	− 0·024	0	−0·001	0	−0·001	− 0·039	0·0001
			− 0·252	0	− 0·007	0	0·001	0	0·010	0	0·003	− 0·248	
(20)	$V_7 = 0·00003$	$V_7 = − 0·00772$	0	0	0	0	0	0	−0·010	0	0	− 0·010	0
			− 0·252	0	− 0·007	0	0·001	0	0	0	0·003	− 0·258	
(21)	$V_1 = − 0·00059$	$V_1 = 2·50436$	0·250	0	0·010	0	0·002	0	0·001	0	0·001	0·265	−0·0016
			− 0·002	0	0·003	0	0·003	0	0·001	0	0·004	0·007	$H_L = 6·2898 \times 10^6$
(22)	$H_L.v$ effect of V_1		− 15·601	0	− 0·301	0	− 0·057	0	−0·020	0	−0·009	15·988	
(23)	,, ,, V_3		0·252	0	3·535	0	0·087	0	0·019	0	0·008	3·901	
(24)	,, ,, V_5		0·008	0	0·011	0	0·263	0	0·007	0	0·002	0·291	
(25)	,, ,, V_7		0·001	0	0·001	0	0·002	0	0·051	0	0·001	0·056	
(26)	,, ,, V_9		0	0	0	0	0	0	0·001	0	0·016	0·017	
(27)	Sum		− 15·342	0	3·249	0	0·298	0	0·059	0	0·022	− 11·716	
(28)	$H_L.v$ effect accounted for		− 15·380	0	3·423	0	0·292	0	0·056	0	0·016	− 11·594	
(29)	Residual $M_L = 10^6 \times$		0·038	0	− 0·174	0	0·006	0	0·003	0	0·006	− 0·122	$H_L = 6·2898 \times 10^6$

and thus equation (41) reduces to

$$10^9 - 2716000(A_{1,1} + B_{1,1})V_1{}^2 = 10 \cdot 5 \times 10^6 F_{1,1} . V_1,$$

or to

$$10^3 - 424V_1 - 2 \cdot 69V_1{}^2 = 0, \text{ by Tables VI and VII.}$$

The positive root of this equation is given by

$$V_1 = 2 \cdot 415 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (46)$$

The large negative root is plainly inadmissible.

20. Equation (46) gives a suitable first operation for the relaxation process (exhibited in Table VIII). This may be summarized as follows :—
First, using in line 1 the operation suggested by (46), it is found that this liquidates the greater part of $(M_L)_1$ at the cost of introducing other " harmonics ", fairly small in amplitude. The resulting change in H_L is also recorded in its appropriate column (column 13)—namely,

$$\varDelta H_L = 2 \cdot 415 \times D_1 = 6 \cdot 5586 \times 10^6 \text{ lb., from Table VII.}$$

Next the greatest " residual " is liquidated (in this case $(M_L)_3$) by an appropriate displacement (V_3) chosen with the aid of Table VII, and this process is repeated until $(M_L)_1$ has been reduced to $15 \cdot 323$ and the other residuals are all negative and small. At this stage (that is, after six operations) the $\varDelta H_L$'s are summed to obtain $H_L = 6 \cdot 2894 \times 10^6$ lb., as shown in the last column of the addition following line 6.

H_L is required in order that allowance may be made for the terms in (41) which have been neglected hitherto. The quantity

$$- H \ \Sigma_n[(A_{\lambda,n} + B_{\lambda,n})V_n],$$

taken with due regard to sign, evidently represents an addition to the " residual " $(M_L)_\lambda$: that is to say, any one displacement V_n will entail, by reason of the terms so far neglected, additions to the first, second, . . . residuals given by

$$- H_LV_n \times [(A_{1n} + B_{1n}), \ (A_{2n} + B_{2n}), \ . \ . \ . \ \text{etc.}], \ . \ . \ (47)$$

and these, when H_L is known, can be calculated from Table VI.

Lines 7 to 11 of Table VIII are derived in this manner. Thus the figures in line 7 represent the quantities (47) when $n = 1$, V_1 has its value $2 \cdot 5129$ as recorded in column 2, and H_L has its value $6 \cdot 2894$ as recorded in column 13. The figures in lines 8–11 are found similarly, and in line 12, by addition, correct values of the residuals are obtained which result from the V's recorded in column 2. A line drawn right across the Table and immediately above line 12 serves to indicate that with the data given in line 12 computation, in effect, starts afresh.

21. Lines 13–21 relate to further " relaxations " performed in the manner of lines 1–6, and the summation after line 21 gives, in their

appropriate columns, the new value of H_L and new values for the residuals. The latter have already been partially corrected for "$H_L \cdot v$ effect" in lines 7–11 : to complete the correction (in lines 22–26) additions are made representing (47) for the new values of H_L and of the V's as recorded in column (2) ; but after a further summation the corrections made previously in lines 7–11 are subtracted (line 28). Thus the figures in line 28 are summations of the figures in lines 7–11, and line 29 gives the result of subtracting line 28 from line 27. Again a line is drawn right across the Table, immediately above line 29, to indicate that with the data there given computation would start afresh.

Actually the residuals in line 29 are so small in relation to the initial moment $(M_L)_1 = 1{,}000$ that the solution can be accepted. The displacement components V_1, V_3, . . . V_9 which correspond with this initial moment have been found, and this result is recorded in the first line of Table IX. Similar calculations, not reproduced here, have led to the figures given in the other eight lines of the Table.

The calculations are simpler as regards the "even harmonics" $(M_L)_2$, $(M_L)_4$, . . . etc., for the reason that the "$H_L \cdot v$ effect" is no longer present. (This can be seen from Table VII, where D_2, D_4, . . . etc., are zero.)

22. In section 9, v was defined as the vertical displacement of cable and girder from their "dead-load configurations", and in (26), $v \operatorname{sech}^2 \lambda\!\left(x - \dfrac{L}{2}\right)$ was expressed as a series of harmonic terms involving V_1, V_2, . . . etc. Thus v can be deduced from Table IX.

Of more importance in practice is the partition of load between cable and girder. The moment sustained by the girder is easily determined ; for in (41) the girder contributes to $(M_L)_\lambda$ by the term

$$(M_G)_\lambda = \lambda^2 \pi^2 \frac{B}{L^2} \Sigma_n [B_{\lambda,\,n} V_n], \quad . \quad . \quad . \quad . \quad . \quad (48)$$

and thus, when the V's are known, $(M_G)_\lambda$ can be deduced from Table II.

23. Because the M's are not linear functions of the V's, it is not possible by simple multiplication to deduce the V's for other values of the applied bending moments. V's calculated on this basis will, however, constitute a very good starting assumption for a process of relaxation whereby they can be modified to make the solution "correct." Calculations made in this manner have led to the results shown in *Figs. 3–5* (pp. 310–311), where the abscissæ give the multiplying factors μ_n and η by which V_n and H_L as deduced from Table IX on the basis of simple proportionality must be increased. Both factors, of course, depend upon the nature of the applied moment : *Figs. 3, 4* and *5* give their values as related with $(M_L)_1$, $(M_L)_3$ and $(M_L)_5$ respectively. It will be seen that in every case the factor is represented closely by a linear function of the M.

TABLE IX.—THE ADDITIONAL DEFLEXION-COMPONENTS (V_n's) AND THE ADDITIONAL HORIZONTAL COMPONENT OF CABLE STRESS (H_L) RESULTING FROM HARMONIC LOADINGS $(M_L)\lambda = 10^9 \sin \lambda \pi \frac{x}{L}$ LB.-FEET (V_n's IN FEET; H_L IN LB. WEIGHT)

| λ | Values of V_n | | | | | | | | | $H_L \times 10^{-6}$ lb. |
	V_1	V_2	V_3	V_4	V_5	V_6	V_7	V_8	V_9	
1	2·50517	0	−0·53930	0	−0·03968	0	−0·00770	0	−0·00236	6·2899
2	0	27·24455	0	−0·73514	0	−0·14141	0	−0·05375	0	0
3	−4·55017	0	14·39767	0	−0·39659	0	−0·07848	0	−0·03078	0·4212
4	0	−0·29523	0	8·71000	0	−0·25578	0	−0·05446	0	0
5	−1·03251	0	−0·16523	0	5·76862	0	−0·17061	0	−0·03672	0·1022
6	0	−0·03550	0	−0·12418	0	4·08381	0	−0·12361	0	0
7	−0·38569	0	−0·01610	0	−0·09102	0	3·03601	0	−0·09216	0·0386
8	0	−0·01035	0	−0·01573	0	−0·07171	0	2·34021	0	0
9	−0·18800	0	−0·00328	0	−0·01194	0	−0·05684	0	1·85900	0·0188

Fig. 3.

$(M_L)_1$ FACTORS.
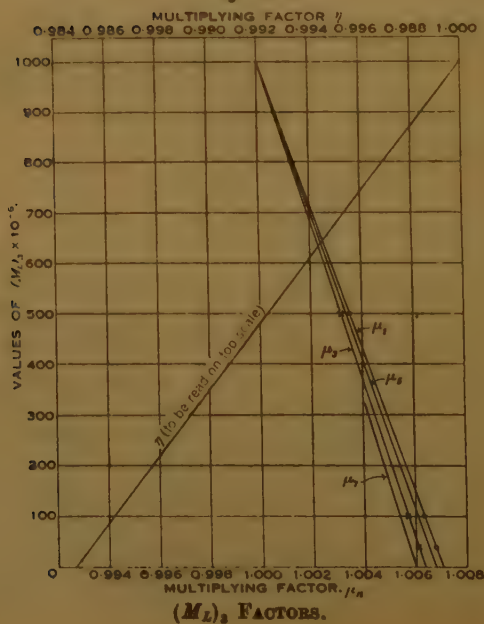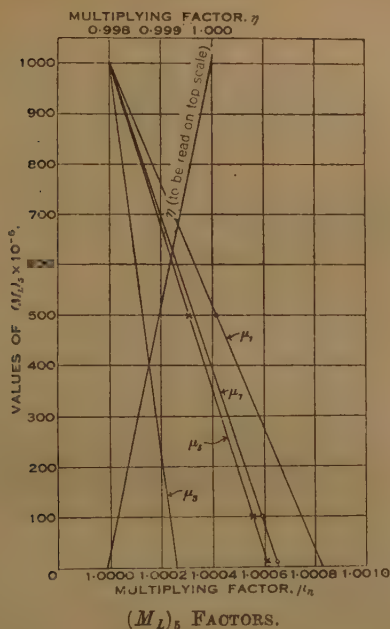
Fig. 4.

$(M_L)_2$ FACTORS.

Fig. 5.

$(M_L)_5$ FACTORS.
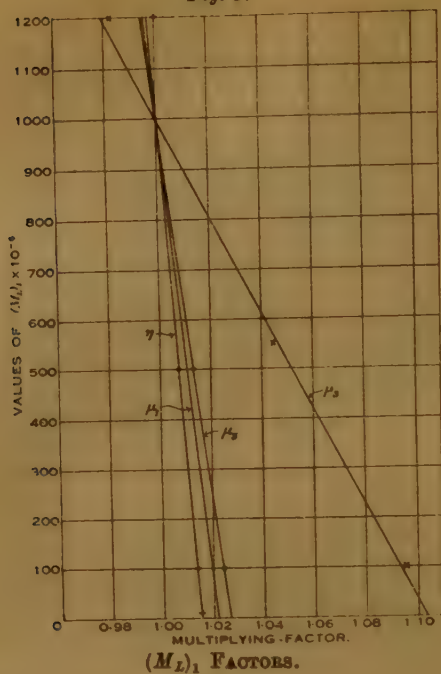
24. For the same reason correct results will not be obtained for a specified loading by superposing the V's found from Table IX for the component M's considered severally. The V's so calculated, however, and the "H_L" which corresponds with them according to (40) and Table VII, will constitute a very good starting point for a relaxation process, conducted on the lines of Table VIII, by which they can be modified to give a "correct" solution.

25. To summarize this second part of the investigation:—In Tables I–IV, values appropriate to any truss having a dip/span ratio of 0·1 have been given to four quantities $(A_{\lambda,n}, B_{\lambda,n}, C_{\lambda,n}, D_n)$ which occur in the relations between loading, deflexion, and cable-tension. Values appropriate to other dip/span ratios are easily calculable. For a particular truss (representative of the Manhattan suspension-bridge) the Authors have calculated appropriate values of the derived quantities $-H \cdot F_{\lambda,n}$, D_n (Table VII), and they have used the relaxation procedure to deduce the deflexions which result from particular (harmonic) loadings. Similar methods can be applied without difficulty to any particular truss. Finally, they have examined the extent to which superposition is permissible, and they have indicated a procedure whereby, when estimates have been obtained by superposition, the resulting errors can be corrected. Applied systematically, their methods enable a designer to examine, quickly and

accurately, the effects of any loading which may be imposed upon a contemplated truss.

In this Paper (to focus attention on principles) conditions of simple support have been assumed for the girder. By obvious modifications it can be extended to other terminal conditions, and little more difficulty is presented by a girder of which the flexural rigidity varies along its length.

This Paper is accompanied by five sheets of diagrams, from which the Figures in the text have been prepared.

Discussion.

Professor Southwell observed that the principles that underlay the type of construction known as the stiffened suspension truss were as follows. Since the cable, although it was sufficiently strong to support the load on the bridge, was not sufficiently stiff to prevent excessive movement of the decking, a girder was necessary to carry the latter ; vertical connexions (the suspension-rods) had to unite the cable and the girder, the cable taking the major share of the load whilst the girder acted as a distributor. That truss, taken as a whole, was a redundant and not a simple structure ; like any redundant structure, it was capable of being self-strained. Since it was redundant, statical considerations did not enable the proportions of the given load taken by the girder and by the cable to be ascertained, yet without that knowledge the dimensions of the structure could not be settled without risk either of danger on the one side or of waste of material on the other. That, stripped to its essentials, was the problem of the stiffened suspension-bridge.

Certain simplifying assumptions had been made in the Paper. For example, the vertical suspension-rods, since they were loaded, would stretch ; but it would complicate the analysis and bring no compensating advantage if that factor were taken into account in a first treatment, so the Authors had assumed that the vertical deflexions of the girder and of the cable were the same. Again, the ends of the girder might be fixed, but they were regarded in the Paper as simply supported ; the cross section of the girder might (and for economy of material presumably should) vary from point to point along its length, but in the Paper the girder was treated as being of uniform cross section. All the essentials of the problem were shown in *Fig. 1* (p. 290), without any of the inessentials.

With great diffidence (for a cautious man did not readily differ from Professor S. Timoshenko) the Authors believed that they had detected an error in Professor Timoshenko's treatment of the problem[1]. Professor Timoshenko took no account in his analysis of the horizontal component of displacement that was bound to accompany the vertical displacement, of which he did take account ; yet if the horizontal displacement, u, were

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[1] Footnote (¹), p. 290.

neglected, then the resulting tension in the cable did not satisfy the con-
dition that its horizontal component had to be uniform. Professor
Timoshenko apparently missed that point for a very curious reason,
namely because he derived his expression for the cable-tension by an
ingenious energy-proof. Assuming that points on the cable came down
purely vertically when the live load was applied, he calculated the work
done by the load in consequence of that descent, and equated that work
done to the total increase of the strain-energy stored in the cable. That,
at first sight seemed satisfactory, and it was easy in imagination to put
in frictionless constraints which would compel the cable to come down
purely vertically : if they were frictionless constraints they would do no
work, so that no work would be done against them, and they would not
enter into the energy-equation. Nevertheless, those constraints would
have to exert horizontal forces, and those horizontal forces would impair
the constancy of H. That was the Authors' reason for differing from
Professor Timoshenko. The result of the Authors' analysis of the move-
ment of the cable was equation (11) (p. 295) including a term involving u,
the horizontal displacement. Moreover, u also appeared in equations (6)
and (7) (p. 294), which related to the increment of cable-tension due to
dead load, and it greatly complicated the analysis.

The Paper then dealt with the quantitative aspect of the movement.
The formulas would have been very complicated if the cable form were
assumed to be a parabola, but it was relatively simple if it were assumed
to be a catenary : for ordinary dip/span ratios the two curves could hardly
be distinguished, so the Authors had assumed the catenary form for
simplicity instead of the parabola. They also assumed that the live
loading which was to be investigated had been represented by its associated
bending moment according to the rules of statics, and so they did not refer
to a live loading w_L but to a bending moment M_L, to be taken jointly by
the cable and the girder. They then proceeded to study the effect on M_C
and on M_G of each harmonic component of displacement. The effect of
the terms involving u (neglected in Timoshenko's analysis) were found to
be important only in relation to the first, second, and possibly the third
harmonics of the curve of deflexion, but were nevertheless important,
because it was precisely those harmonics that usually predominated.
Equation (39) (p. 304), giving the effect of the nth harmonic of displace-
ment, V_n, on the live-load moment, M_L, contained a factor H_L denoting
the increase in the cable-tension which resulted from the deflexions, and
itself given by equation (40). Every deflexion not only altered the shape
of the cable on which the cable-tension could do its work, but also altered
the cable tension which was to do the work. In consequence superposition
was not applicable, and a result found for a particular magnitude of bending
moment could not simply be doubled or trebled to find the effects of twice
or three times that bending moment. The non-linear form of equation

(39) was the essence of the difficulty, both in Professor Timoshenko's analysis and in that of the Authors.

Even if the horizontal displacement were purely sinusoidal, according to equation (39) the bending moment would not itself be strictly sinusoidal : a second-order term came in on account of H_L, and therefore a solution could be found only in relation to some particular bending moment.   So before numerical results could be obtained it was necessary to have values for the size of the cable and for the stiffness of the girder : in section 18 (p. 305) of the Paper the Authors had adopted figures which were generally representative of the Manhattan bridge.   It was shown that a single harmonic of M_L entailed first, third, fifth, seventh, and ninth harmonics in the displacement.   Calculations had been done for nine harmonic forms of M_L, so that if the values of M_L could be simply multiplied then the work would have been finished : since, however, Hooke's law did not apply to the problem, supplementary calculations had to be made for different values of M_L.   *Figs. 3, 4,* and *5* (pp. 310–311) showed how much the results had to be multiplied for different values of the bending moment : if Hooke's law had applied, the multiplying factor would have been 1 for every value of the bending moment, and all the lines shown would have been vertical. What was unexpected and fortunate, was that they were very nearly straight lines ; the Authors had calculated three points on each curve, and a straight line was a sufficiently accurate representation in each case. Given those three diagrams plus the Tables, a designer could deal with simple harmonic bending moments of any magnitude.

The last point to which Professor Southwell wished to refer was the question of superposing different loadings.   As a first approximation it might be assumed that the principle of superposition did apply to the problem : thereby allowance would be made for a large part of the bending moment, but a part would still be unaccounted for ; one application of the relaxation process, however, would then be sufficient to account for the remainder of the bending moment.   The Authors had not given an example of that further process, because it was so much like Table VIII (facing p. 307) ; but Mr. Atkinson had specimen Tables, and if necessary they could supply one to be included in their reply to the Discussion.   The Authors did not think, however, that it was necessary.   As stated in section 24 (p. 311), the assumption of superposition, although not correct, afforded a very good starting point for a relaxation process, conducted on the lines of Table VIII, by which final corrections might be determined.

Professor A. J. S. Pippard remarked that the essential difficulty in the whole analysis of the suspension-bridge problem was, as the Authors had pointed out, the non-proportionality of load and displacement.   If it were not for that, the ordinary methods of structural analysis could be applied, and with a certain amount of hard but perfectly straightforward work of the usual type it would be possible to obtain an exact solution

for any conditions of loading for any type of structure of the kind i i
question which the engineer cared to use. That was always done in text
books, and was known as the elastic theory of the suspension-bridge
That theory involved a great many troublesome points when criticall
examined, and it was only by the work of Professor J. Melan, and c
a number of American engineers and scientists, that the matter had bee
put on a rational basis. The latest work was by Professor Timoshenko
to which the Authors had referred. The result of the more exact analysi
was to show a great economy in material in long-span bridges.

From the theoretical point of view the value of the present Paper lay
in the fact that a term neglected by Professor Timoshenko, namely the
horizontal displacement of the cable, had been introduced and dealt with
As Professor Southwell had pointed out, however, the theory was still
incomplete, and Professor Pippard was not at all sure that it was quite
legitimate to assume the hangers to be inextensible; he would like to
know the effect of that neglect compared with the effect of neglecting the
value of u, the horizontal cable-displacement. He felt that they were
comparable in magnitude, and that if it were necessary to take u into
account it would also be necessary to take account of the extension of the
hangers. Perhaps something had been done on that, but it was not
included in the Paper.

One unfortunate omission in the Paper was that no numerical value
was given of the effect that u had upon, say, the cable-tension. Whether
or not it was worth taking the effect of u into account would depend a
great deal upon the numerical value. It was possible that the Authors
had made that calculation and could give some actual values; they had
considered as an example the Manhattan bridge, which had also been
dealt with by Messrs. Johnson, Bryan, and Turneaure and by Professor
Timoshenko. It would therefore have added greatly to the value of the
present Paper if the tension due to live load could have been included.

Professor Pippard did not like the Authors' definition of the term M_C,
which was referred to (p. 296) as that part of M_L which was transmitted
from the girder to the cable; again, in Professor Southwell's introductory
remarks words were used which seemed to imply that a bending moment
was transmitted to the cable. One of the assumptions was, however, that
the cable was flexible, and the essence of a flexible cable was that it could
not take any bending moment. It was unfortunate, therefore, that the
definition of M_C had been given in those terms; and he thought that it
would have been better to have defined M_C as the relief given to the bending
moment which would ordinarily be carried by the girder due to the trans-
ference of load from the girder to the cable. He was not criticizing the
mathematics in any way, and the Authors' method was convenient, but
the statement to which he referred might have been made clearer to the
engineer.

The Authors had taken the catenary as representing the cable-shape, and that was perfectly justifiable. In actual fact—it might be of interest to have opinions on the point from those who had designed suspension-bridges—he was not sure whether the curve used was a catenary, a parabola, or something between the two. It was true that the two curves were very close, but he would like to know the actual effect of the horizontal displacement u, in order to see whether or not it might not be entirely swamped even by small differences between the actual shape of the cable as erected and the assumed shape.

One point which was not clear was the reference throughout the Paper to the bending moments and their expression as Fourier series. Professor Southwell was probably a little optimistic in suggesting that it was an everyday performance to do Fourier series, and the value of the Paper would be increased if the Authors would include a note to explain that the first step was the drawing of the bending-moment diagram from the given live loading, and the next to express that bending moment as a Fourier series.

There was one point which he did not follow completely in Professor Southwell's demonstration of the constancy of H. Professor Southwell gave the impression that the constancy of the horizontal component of tension was dependent upon the elastic stretch of the cable. Even if the cable were inextensible, whilst remaining flexible, there would still be a change in the horizontal tension when a live load was put on the bridge.

Professor Southwell and his pupils had in the last few years given a great deal of time and thought to the development of the method of relaxation, but so far as Professor Pippard was aware the present Paper was the first time that it had been applied to the problem of a structure which did not obey Hooke's law, and from that point of view the work marked a stage in the development of the theory of structures.

Mr. J. S. Wilson remarked that the Authors had made allowance for probably more of the disturbing influences than had been done in any of the earlier mathematical investigations ; although the solutions seemed almost too complex for application, the Authors had overcome the difficulty by showing how the " relaxation " method made it simple. An engineer designing a stiffened suspension-bridge had to visualize and appreciate the actual conditions, and compare them with those assumed in any theoretical treatment the results of which he proposed to rely on in his design. Some of the conditions, about the existence of which the engineer could have no doubt and which were ignored or only partly taken into account in mathematical treatment, were :

With the live load on the bridge the whole length of the ropes between the anchorages would stretch and the ropes would either move on the tower-saddles or the towers would be allowed to rock slightly, the points of suspension not being fixed in the mathematical sense.

The modulus of elasticity of the main suspension elements would not be the same as that of the stiffening girder. In comparatively small bridges—Mr. Wilson had not been concerned with any very large ones—the ropes were made up of cables bunched together, and the modulus of elasticity of those cables was comparatively low. In the case of some recent bridges the cables had been pre-stressed in order to obtain a higher modulus and to make it more or less constant, but even so it was comparatively low and could vary considerably.

The hangers were spaced at appreciable distances apart, and the rope took up a polygonal shape rather than a smooth curve. They were often made of cables with a modulus probably lower than that of the main cables, having regard to their end connexions, and were of appreciable length; under live load they would certainly stretch. They were often fitted with screwed couplings for adjustment, and any want of accuracy in their adjustment would affect the stiffening girder.

Such influences might or might not have a big effect, and in the Paper the Authors demonstrated that certain effects were either negligible or could be accounted for.

Mr. Wilson had used the Résal [1] method of design. It had the great advantage that the results could be shown graphically. Résal regarded the problem as similar to that of an inverted arch. Applied to the stiffened suspension-bridge problem the process consisted of drawing the polygon for the loads which were the sums of the dead and unevenly distributed live load at each point. The exact length of the polygon between the points of suspension was calculated and then a polygon of that length to suit the dead loads was drawn. The bending moment at any point in the stiffening girder was the product of the horizontal component of the cable-tension under combined dead and live load and the intercept between the two polygons at the point. In developing his mathematical investigation Résal considered the influences of the errors introduced by the various assumptions, and was satisfied that the method was substantially accurate for the proportions and conditions met with in practice.

Mr. Wilson had found that for specific cases the bending moment obtained by the Résal method agreed very closely with that derived from Max am Ende's equations [2].

It would be very helpful if the Authors would examine the Résal method, for it was direct and not too difficult to apply to practical cases.

As it appeared to be impracticable to solve the problem mathematically if allowances were made for all the serious disturbing factors, the situation pointed to the need of carefully made experiments on a model or a small

[1] G. L. Le Cocq, "Ponts Suspendus," vol. i (1911); Jean Résal, "Cours De Ponts Metalliques," vol. ii (1912).

[2] "Suspension Bridges with Stiffening Girders." Minutes of Proceedings Inst. C.E., vol. cxxxvii (1898–99, Part III), p. 306.

bridge in which all the factors would have effect and could be varied.   It would then be possible to establish the accuracy, for practical application, of the various approximate methods.

Miss Letitia Chitty observed that the Authors had deduced equation (11) to replace equation (3), which corresponded to Timoshenko's equation. The Authors were not the only people to criticize Timoshenko's equation; Professor Hans Rode, in his criticism of Timoshenko's Paper, pointed out that the complete equation had not been taken.   Unfortunately, he had not himself given the complete equation in that discussion; he had only given it with the simplification of u and V so connected that the cable was in effect inextensible.   Professor Rode was killed in 1930, and she did not know whether or not he had continued and published the full theory. The Authors said on p. 297 that they could neglect extension in the cable when it was subjected only to dead loading.

Most of her other criticisms of the latter part of the Paper had been answered.   She had been tempted to continue the calculation herself and to analyse harmonically the bending moment corresponding to a load over a quarter of the span, as she would have liked an answer giving a comparison of their results with those of Timoshenko and of Johnson, Bryan, and Turneaure, but she noted that the Authors had assumed a different value for Young's modulus from Timoshenko.

Mr. E. S. Needham said that for a clear picture of the functioning of the stiffening truss under load, he would like briefly to refer to a diagram in a Paper published in the Proceedings of the American Society of Civil Engineers, and to the discussions which had appeared in subsequent issues of the Proceedings throughout 1938 and in January, 1939.   That diagram *Fig. 4* in that Paper [1]) represented the deflexion of the cable and of the stiffening truss under the partial live load which produced the maximum moment at the quarter-point.   The final position of the stiffening truss and cable was shown on that diagram, and it was a compromise between the original dead-load position of the cable and the deflexion-curve of the unstiffened cable.   The maximum vertical distance between the unstiffened-cable curve and the final curve represented the degree of rigidity provided by the stiffening truss.

It would appear that American engineers engaged in the economics of long-span suspension-bridge design had not found the answer to the problem of what degree of rigidity to provide or of how flexible the stiffening trusses might safely be made.   Mr. O. H. Ammann concluded that:

> " both the criteria for rigidity and the factors involved in the design of
> an adequate and economical stiffening system are so complex that an
> attempt to devise formulas or rules of design on a theoretical basis

[1] S. Hardesty and H. E. Wessman, " Preliminary Design of Suspension Bridges." Proc. Am. Soc. C.E., vol. 64 (1938), p. 69.   (January 1938.)

appears fruitless. In smaller and lighter structures the principal object is rigidity against vibrations or oscillations which may be detrimental to the structure as well as objectionable to traffic. As the bridge increases in span and weight, this object becomes less and less important, except in the design of the floor system. In fact, the flexibility of the structure as a whole becomes advantageous, because it tends to reduce local impact effects and vibrations. The principal purpose of limiting deflexions and other deformations of the main carrying system in long spans is to avoid excessive gradients in the roadway and to prevent deformations that may result in excessive local bending, distortions or transverse tilting of the floor structure. Limiting grades may be important in bridges carrying rail traffic, but for modern highway traffic even that object loses significance, because the grade changes that may be actually produced in a well-designed structure are not sufficient to form a serious impediment to traffic."

The trend in American practice was towards greater flexibility in the stiffening truss. The old rule for the appropriate depth-ratio was between $\frac{1}{40}$ and $\frac{1}{60}$ of the span length. The George Washington bridge, of 3,500-foot span, was designed with a truss 29 feet deep, a depth-ratio of only $\frac{1}{120}$. The Golden Gate bridge, with a span of 4,200 feet, had trusses 25 feet deep, a ratio of $\frac{1}{168}$. The latest development was in the design of the Bronx-Whitestone bridge in New York, with a centre span of 2,300 feet, which was stiffened by plate girders 11 feet deep, a ratio of $\frac{1}{210}$. The George Washington bridge was designed with double deck, but up to the present time the railway deck had not been constructed, and the road traffic was carried on a completely unstiffened deck. That radical departure from conventional practice was permissible because of the enormous dead load in the cables. It would appear, therefore, that the choice of a stiffening system was based on experience, judgement, and economy. It had to be remembered that the stiffening truss usually served also as the boom or chord member for the horizontal transverse wind-bracing system. In the case of the Golden Gate bridge stiffening trusses, the stress due to live load was only 30 per cent. of the total stress, and the remaining 70 per cent. was due to transverse wind load.

Having chosen in a preliminary design the characteristics of the stiffening truss, the final calculations of the shears and bending moments entered the field of higher mathematics of the type described by the Authors, which he would not attempt to discuss. It was generally assumed that the maximum moment in the stiffening truss occurred at the quarter-point, but Mr. L. S. Moisseiff had pointed out, in the discussion of the Paper referred to, that that was not the case in long-span bridges. In the Golden Gate bridge Moisseiff had found the greatest positive moment to be 10 per

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[1] Footnote ([1]), p. 319.

cent. greater at one-eighth of the span-length from the tower. In the Whitestone bridge the maximum moment was found to be 23 per cent. greater at one-tenth of the span-length from the tower. If the Authors had carried their investigations as far as determining the point of maximum bending, it would be of interest to know whether their results confirmed that discovery.

The Chairman made certain remarks of a preliminary nature, but asked that they should not be published, as he wished to amplify them in writing[1].

Professor Southwell, replying first to Professor Pippard, said that Mr. Atkinson had done the detailed work and could probably give a good deal of the information asked for. With regard to the constancy of the horizontal tension H, really there were two questions involved. First there was the question whether or not the tension remained constant (that was, uniform) throughout its whole length : it always did if the loading were vertical. Then there was the question whether or not the horizontal component of tension remained constant throughout time ; it did not if the loading were altered.

On the question of the use of M instead of w, he pleaded the excuse that he had followed Professor Inglis : it was his association with Professor Inglis on the Bridge Stress Committee that first made him realize that it was better to talk about the bending-moment diagram associated with a given transverse loading than about the loading itself ; a far better sense of what was structurally important was obtained by looking at the bending-moment diagram than by looking at the loading curve. Again, through that association he had come to think of the drawing of a bending-moment diagram and of its analysis into harmonic components (the Fourier series was hardly in question) as almost a routine matter.

As for the assumption of a catenary for the cable form, he thought that merely drawing out a catenary and parabola of practical dip/span ratios would show satisfactorily that there was no way in which that difference would be sensible. Perhaps Mr. Atkinson would investigate the matter and given numbers, but Professor Southwell thought it permissible to trust his intuition in that matter. It was otherwise with the question of " u-effect."

Regarding the definition of M_C as bending-moment transmitted to the cable, that again turned on the use of M to represent w. It was true that actually it was loading that was transmitted to the cable, but it was convenient to represent that loading by its associated bending moment M_C. Accepting Professor Pippard's definition as more precise, he still ventured to differ slightly from Professor Pippard's view : he would say that an

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[1] Professor Inglis's remarks will be published with the Correspondence on the Paper.—SEC. INST. C.E.

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ordinary cable hanging in a catenary did sustain a bending moment, in virtue of its shape and of the horizontal forces applied to its ends; it sustained the bending moment which was necessarily associated with the vertical load, just as effectively as a girder. That, however, was only a matter of terms.

The omission of numerical values for the effect of u had been remarked with regret. The Paper was completed last summer, and since that time Mr. Atkinson had been out of Professor Southwell's reach; but he was continuing the work for a thesis, and would doubtless be able to produce some illustrative results. It was not possible, however, to quote definite numbers, because of the failure of proportionality: a different figure held in every case, and it would not be possible to do more than give one or two illustrations.

Professor A. J. S. Pippard, interposing, said that he had been thinking of the case of the Manhattan bridge.

Professor Southwell said that even for that one bridge a different figure would apply not only to each type of loading, but to every magnitude of a given type. He thought the best thing to do would be to show the difference between H_L as given by Timoshenko's formula and H_L as given by their own; but as a rough indication Mr. Atkinson had told him that evening that he had made a calculation which showed that the omission of the u-term made a difference of about 19 per cent. Professor Southwell did not find that result surprising, in view of the enormous extensions which would be entailed by purely vertical displacements.

In regard to the neglect of extensibility in the hangers, he thought its nature could be seen by intuition; the hangers could be extended elastically only by a very small fraction (less than $\frac{1}{1000}$) of their length and the consequence of the extension would be that the girder would go down farther, but only slightly farther, than the cable; therefore a little more load would come on the girder than would come on it if the hangers were rigid, as the Paper had assumed (the amount would be easy to investigate by relaxation methods).

He agreed that the mathematics that had been required were regrettably complicated, but the Authors had done that part once for all, and nothing more was required of the designer in relation to future calculations than a mere substitution of numerical values in the resulting formulas.

Mr. Wilson had raised three points to which answers were required. The Authors did not suggest that harmonic components should be used to represent the curve of live loading, but to represent the bending-moment diagram (which was, of course, the curve into which a flexible catenary would hang under that loading). It was an entirely different matter to turn a curve of that sort, without steps or sharp angles, into a number of harmonic components (for example, even without calculation, it could be seen fairly clearly how much would be accounted for by the first component).

The construction of a bending-moment diagram corresponding with a given system of loads was a problem familiar nowadays to every engineering student : when that had been done, the resulting curve yielded very rapidly to harmonic analysis.

Another point related to the effects of guy wires which permitted some yielding of the cable-supports, of finite spacing and extensibility of the suspension-rods (extensibility of the cable, however, had been taken into account by the Authors), and so on. Those were practical complications making for less simple conditions than had been assumed in the Paper ; he was confident that all could be dealt with (the value of Young's modulus for the cable had, however, to be known), but it was evident in advance that they would make the problem harder and that in consequence the mathematics would be even more complicated. The same question would arise if and when they came to investigate the accuracy of Résal's method. If the Authors were to test whether the method was accurate, they would have to bring it up against an accurate standard of comparison—such as they hoped their own work provided : it would be rather hard if in the end they were again told that their results were unsuitable for practical engineers.

Mr. Needham's contribution had gone outside his own knowledge and experience and beyond the scope of the Paper, but its concluding remarks had gone to strengthen his conviction that the question to which the Authors had addressed themselves would arise in the designer's mind whatever else was there as well. There would be a great deal more in the designer's mind, but the question of how a simple truss behaved, and of what was the partition of load between girder and cable, was one which could not be avoided. Mr. Needham had given a rule from an American Paper which seemed to be much the same as that which Mr. Wilson had put forward, and Professor Southwell could only repeat that his intuition failed to tell him whether it was even approximately correct, although he could well believe that it might be so.

He had dealt already with the suggestion that account ought to be taken of the extension of the suspension-rods ; to the similar suggestion regarding the horizontal displacement, u, of the girder itself he would reply that if it were necessary to examine the effect of horizontal displacement, u, in a girder forming part of a suspension-truss, then it would be necessary to do so for every girder, whether under a cable or not. Reasons could be seen why u on the girder would be free to occur and why nothing serious would happen when it did : but, on the other hand, reasons could be seen why suppression of u in the cable would make a very big difference to the stretch of the cable, and therefore a big difference to the tension in the cable. Professor Southwell would not be prepared to recommend Mr. Atkinson to look into the question of the u effect for the girder, because he felt certain in advance that that would have no appreciable effect on the

question of partition. It was just a question of how the girder behaved under what it had to do for itself.

He would sum up by saying that he saw the problem as followed : the physical problem could not be made any simpler than the Paper had made it, without cutting out something essential. If it were thought that the Authors had made it too simple, he could only reply that the present Paper was a first step, and that anything more taken into the basic analysis would have made the mathematics even worse. There was no doubt whatever that loads could not be superposed, and to that extent new methods were necessary, as was evidenced by the number of new methods (including Professor Timoshenko's) which had been proposed for exactly the problem with which the Authors were dealing. By showing what adequate treatment of the single span entailed, and by showing that, although it entailed a great deal of basic analysis, the resulting equations proved to be tractable by the relaxation method, he felt that the Authors had put themselves and others interested in a position from which they might go on with confidence to other problems, such as those of two or three spans, and so on.

In completing his reply in writing, Professor Southwell pointed out that although the point was made in his opening remarks, he was not sure whether or not all those taking part in the discussion had fully appreciated that the simplified problem which was treated in the Paper was identical with what had been treated in Professor Timoshenko's, and that both Papers were alike in their use of harmonic components as a representation of specified loading actions. If the Authors had been able to accept Professor Timoshenko's analysis in its entirety, then the only purpose which a new Paper could have served would have been to indicate the utility of the relaxation method as an alternative to Professor Timoshenko's detailed treatment : equation (3) would have been the starting point, and section 17 would have become section 1. Actually the question of " u-effects " presented itself, and necessitated the lengthy mathematical investigation which formed the first part of the Paper. That resulted in an equation similar to Professor Timoshenko's equation (3), but different in that it contained additional terms involving u : the second part of the Paper discussed that modified equation, but it might equally have dealt with (3).

He had been asked whether or not he had spoken of the Authors' investigation to Professor Timoshenko : he had done so when he had met him last September, and Professor Timoshenko's reply had been what Professor Southwell had expected from his long acquaintance with him : he did not dismiss " u-effects " as negligible in comparison with other effects which both treatments neglected, nor did he suggest that the problem did not call for the introduction of differential equations ; he said " I never mind being confronted with a mistake, provided that I made it not less than 5 years ago."

It seemed to Professor Southwell that problems had to be taken as they were found, and any method was appropriate that gave promise of success ; if later it could be replaced by something simpler, so much the better ; but it was of no use to start from a conviction that every structure found useful in civil engineering ought to behave like an arch or framework and to satisfy Hooke's law, if in fact it were known that the structure in question did not so behave. Further, Professor Southwell believed that appeal to intuition was justified as a means to simplification, but only if the investigator had confidence in his intuition, and not merely on the ground that "it gets one further on." Professor Southwell felt intuitively that the partition between cable and girder would be very nearly the same whether the number of the suspension-rods was very large (as assumed in the Paper) or as small as twenty : therefore in that respect he thought it justifiable to make the more convenient assumption. He had, however, no trust in his ability to foretell by intuition how much extra tension would be thrown on the cable (allowing for its extensibility) by a live load covering, say, half the span ; therefore he wanted to see that question brought to the test of exact calculation.

Some speakers in the Discussion seemed to feel intuitively that neglect of the " u-displacements " would not have sensible effects. He could not share that expectation; he had considered the cable-stress entailed by purely vertical displacements, and how easily the cable could avoid that stress by also moving horizontally, and he concluded that account had to be taken of " u-displacements " in the cable itself if its tension (and hence its share of load) were to be estimated correctly. It was there that the " u-effects " were important. The term $\dfrac{d}{dx}\left(u\dfrac{d^2y}{dx^2}\right)$ was included for completeness, but (as he had indicated) its effect was small whether the cable form was a parabola or a catenary. Where u mattered was in its effect on H_L (the extra tension due to live load), as found in equations (6) and (7) : Professor Southwell recognized that that point should have been made more clearly in the Paper.

He did not, however, after reflection find much in the Paper that he would wish to revise. The Authors could have taken more effects into account, but even stripped to its essentials the analysis had entailed mathematics which, like several speakers, they could have wished were more simple (they had, however, been done once for all). They were told that Résal's concentration on practical dip/span ratios was satisfactory to engineers, and they had followed his example in their numerical work. They were told not to worry about lengthy computations, but to proceed step-by-step : that was exactly what the relaxation method did.

Mr. R. J. Atkinson, who also replied, referred to the difference of 19 per cent. which Professor Southwell had mentioned, and said that he had worked out an example by the deflexion method of Johnson, Bryan, and

Turneaure for a live load high up the span, and had also calculated it by the Authors' method. Unfortunately the figures had not been checked, but they had been done on a calculating machine. It was found that at only about the tenth-point of the span was there a difference of 19 per cent. due to u.

He had some figures for the horizontal tension in the cable as calculated by both methods for full-span loaded and half-span loaded. They did get a slightly smaller increase of horizontal tension due to live load over the full span, but to a practical man the difference was hardly worth noticing, being of the order for full-span loading of about 685 to 684. In the example worked out by Johnson, Bryan, and Turneaure there was a maximum deflexion of the order of 11 feet, whereas taking the Authors' example and working it out by the deflexion method, the deflexion came to about 2 feet. He was afraid that he could not at present quote definite figures to show the importance of the "u-effect" and to compare it as between the parabolic form and the catenary.

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Students' Paper No. 946.

"A Survey of the Present Position in Road Transition-Curve Theory."

By DENNIS FRANK ORCHARD, B.Sc. (ENG.), STUD. INST. C.E.

(Ordered by the Council to be published with written discussion.)[1]

TABLE OF CONTENTS.

INTRODUCTION.

THE importance of the provision of transition-curves in all changes in direction of vehicular motion appears to have been first realized by railway engineers, although their early efforts to introduce them were of necessity crude in conception. Even now, in spite of an extensive literature, many road engineers continue to lay out new curves without the provision of any transition at all. Where transition-curves are inserted, they are often planned in an arbitrary and unscientific manner.

It has been abundantly demonstrated in the more recent publications, notably those of Professor F. G. Royal-Dawson, M. Inst. C.E.[2], that the length of a transition-curve is a theoretically determinable quantity, based on certain fundamental and simple dynamic principles first enunciated by Mr. W. H. Shortt, M. Inst. C.E.[3] In spite of this, many engineers still estimate an arbitrary length for the transition-curve, or else base the length on a chosen gradient to lead up to the full superelevation on the outside

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[1] Correspondence on this Paper can be accepted until the 15th June, 1939, and will be published in the Institution Journal for October 1939.—SEC. INST. C.E.

[2] F. G. Royal-Dawson, "Elements of Curve Design for Road, Railway, and Racing Track." London, 1932.
——, "Road Curves for Safe Modern Traffic and How to Set Them Out." London, 1936.

[3] W. H. Shortt, "A Practical Method for the Improvement of Existing Railway-Curves." Minutes of Proceedings Inst. C. E., vol. clxxvi (1908–9, Part II), p. 97.

edge of the road. It is seldom realized that, in the interests of road-l
safety, correct transitioning of curves is as important as the provision of
superelevation. The often-used argument that the improvement of high-
ways is an incentive to speeding, far from being a justifiable excuse for
the non-provision of transition-curves, is in reality a powerful argument
in favour of correct transitioning. It must be remembered that it is a
physical impossibility for a moving road vehicle to change its direction
without tracing out some kind of transition-curve. Thus, if the curve is
laid out in the form of a circular arc the vehicle is bound to leave its correct
traffic-lane, a practice which becomes increasingly prominent and dangerous
as the speed increases.

HISTORICAL REVIEW.

The first serious attention seems to have been paid to the provision of
transition-curves in France and America, in both cases in connexion with
railway-alignment. Paul Adam was, in 1895, the first to draw attention
to the lemniscate in France [1], showing that it was no less simple in appli-
cation than the cubic parabola, but it was not until 1919 that its use for
road-curves was discussed by C. Galatoire-Malegarie [2], who compared the
lemniscate, clothoid, and cubic parabola for road use, gave an example of
the use of the lemniscate, and also considered superelevation. In both
countries the early suggestions comprised no rational method of assessing
the length of the transition ; for example, J. C. Trautwine's method, intro-
duced in America, is not only empirical in its length but also in its contour.
This method was improved by L. C. Jordan [3], who, by fixing a definite rate
of gain of superelevation (2 inches per second) in place of the formerly
practised gradient method, introduced a variability into the length of the
transition according to the speed of the locomotive. A. N. Talbot's [4]
formula also belonged to the class in which a fixed rate of gain of super-
elevation was the governing criterion of the length. Several other American
authorities adopted similar methods, [5,6], whilst some did not even touch
upon the subject of length, [7,8].

It remained for W. H. Shortt in England and J. R. Stephens in America
to introduce the first scientifically accurate method of assessing the

[1] " Emploi de la Lemniscate de Bernouilli dans les Raccordements de Chemins de
Fer." *Annales des Ponts et Chaussées*, vol. x (seventh series, 1895), p. 383.

[2] " Tracés de Routes a Courbure Continue." *Annales des Ponts et Chaussées*,
vol. 1 (ninth series, 1919), p. 332.

[3] " The Practical Railway Spiral." London, 1913.

[4] " The Railway Transition Spiral." New York, third edition, 1901.

[5] T. R. Agg, " The Construction of Roads and Pavements. New York, third
edition, 1924.

[6] G. W. Pickels and C. C. Wiley, " Route Surveying." New York, 1930.

[7] H. C. Ives, " Highway Curves." New York, 1929.

[8] W. W. Crosby and G. E. Goodwin, " Highway Location and Surveying."
Chicago, 1929.

length of a transition-curve. However, a really lasting basis for the delineation of the curve had still to be provided. In 1925–6 one of the first really serious efforts towards the provision of road transition-curves was made in England on the Great North road in the County of Rutland, and was described by Mr. Henry Criswell, M. Inst. C.E.[1] This was indeed a good step forward, but the curves were still far from ideal ; no attempt was made to determine their lengths on a mathematical basis, and the limitations of the cubic parabola were still present. Another excellent example is provided by the curve-alignments on the Redclyffe Road near Manchester, described by Mr. E. L. Leeming, M. Inst. C.E.[2] In this case the lemniscate form of transition was adopted, but the length again appears to have been arbitrarily chosen, it merely being stated that the commencement of the curve was " about " 210 feet from the centre of the bend.

Even to that date the literature on the subject was of rather limited scope, but a great advance came with the publication of Professor Royal-Dawson's treatise " Elements of Curve Design "[3] and subsequent articles by Mr. Henry Criswell.[4] Professor Royal-Dawson's book at last put the subject on a complete mathematical basis. The length was now a calculated quantity, whilst the cubic parabola and allied curves had at last been abandoned for the much more suitable contour of the lemniscate. Mr. Criswell had by this time also adopted Shortt's standard rate of turning, but preferred the true spiral (or clothoid) to the lemniscate advocated by Professor Royal-Dawson. This literature has recently been further augmented by Professor Royal-Dawson's recent publication, " Road Curves." With the advent of these publications fresh attention has been given to the question of the maximum safe rate of gain of centripetal acceleration. Mr. Criswell has accepted Shortt's formula without modification, whilst Professor Royal-Dawson has adopted it for all speeds above 30 miles per hour and has provided a " graduated empirical formula " for speeds below that value. This empirical formula provides for an increase in the rate of gain of centripetal acceleration for speeds below 30 miles per hour in order to allow for the increased manœuvrability of light vehicles at those speeds. The increase in speed permitted on this basis is 40, 30, 21, 13·5, and 6 per cent. at normal speeds of 5, 10, 15, 20, and 25 miles per hour respectively. Professor R. A. Moyer[5], however,

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[1] " A Simple Treatment of Superelevation, Transition Curves, and Vertical Curves, as used on the Great North Road in the County of Rutland, 1925–26." Public Works Roads and Transport Congress, Final Report, 1929.

[2] " The Superelevation of Highway Curves." Inst. C.E. Selected Engineering Paper No. 50, 1927.

[3] Footnote 2, p. 327.

[4] " Spiral Tables for Highway Design." *Roads and Road Construction*, vol. xiv (1936), pp. 37, 69, 101, 133, 164, 197, 231, 262, 294, 332, 366.

[5] " Skidding Characteristics of Automobile Tires on Roadway Surfaces and their Relation to Highway Safety." Iowa State College of Agriculture and Mechanic Arts, Bulletin 120, 1934.

appears to consider that a rate of gain of centripetal acceleration of 2 feet per second per second in a second is not only safe but desirable even for high-speed work. This contention is further supported by Mr. Frank Hosking[1], of the County Roads Board, Victoria, Australia, who advocates a speed round the curve of 0·8 of the critical speed, where the critical speed is that giving a rate of change of centripetal acceleration of 3 feet per second per second per second, which is the figure commonly taken as being the maximum attainable by the most skilled drivers. On this basis the permissible rate of change of centripetal acceleration is $(0·8)^3 \times 3 = 1·536$ feet per second per second per second; furthermore, Mr. Hosking states that an excess of speed of 25 per cent. over this value is permissible, in which case the rate of change of centripetal acceleration becomes 3 feet per second per second per second, which may be thought a high value even for slow-speed work.

The recent publications on the subject of transition-curves have been devoted largely to the provision of complete Tables for their setting-out. This is a practice which should be adopted with caution, as it is essential that the user of such Tables should be fully conversant with the underlying theory and methods of compiling them.

In this Paper the Author puts forward methods of calculation which are intended to supplement the Tables now available; for this purpose such constants as the maximum permissible rate of gain of centripetal acceleration (hereinafter denoted by C) and maximum permissible centrifugal ratio (hereinafter denoted by B) will be represented by symbols in the final formulas. In this way it will be possible for values of these constants other than those adopted as standard by Professor Royal-Dawson to be used. The Author is of opinion that Professor Royal-Dawson's values are at present the best compromise, but there still appears to be considerable diversity of opinion on this subject, and with the introduction of roads with permanently improved coefficients of friction it may be possible to design for slightly increased values of C and B. Any system of calculation adopting the spiral or the lemniscate must, for the sake of simplicity, make certain approximations; it is proposed as far as possible to give justification for any approximations used.

Available Formulas.

The Cubic Parabola.

The cubic parabola has the simplest mathematical form of all curves which have been used for transitions, and in all cases where the polar deflexion-angle does not exceed about 4 degrees there is little difference between it and the lemniscate or spiral. It may be set out by the co-ordinate method, or the co-ordinates may be used to calculate the polar ray and the

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[1] *Roads and Road Construction*, vol. xiv (1936), p. 180.

polar deflexion-angle. If the curve becomes of such length that it is either undesirable or impossible to set it out by polar rays, it is a simple matter to calculate the chord-lengths round the curve, together with the appropriate polar deflexion-angle. Although it is not proposed to deal with the cubic parabola in detail, a method of tabulation will be given for making the above calculation, as a similar Table is applicable to methods of setting out the spiral and lemniscate to be described later.

TABLE I.

Chord-point.	X	Y	$\tan\alpha$ $=\dfrac{X}{Y}$	α	x	y	x^2	y^2	Individual chord-length or sub-chord.	Total chord-length.
1.										
2.										
3.										
4.										
5.										

The explanation of Table I is given in *Fig. 1*, where 0–1, 1–2, 2–3, etc., represent the chords as actually set out, the chainage being in each case continued from the previous chord-point, whilst the angles are all set out from the origin O. The columns headed X and Y represent the

Fig. 1.
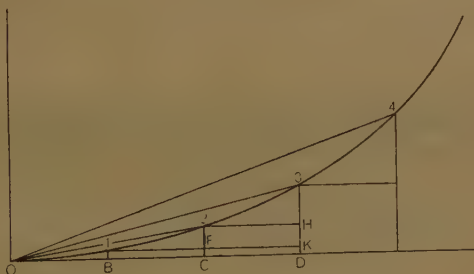
distances OB, OC, OD, B1, C2, D3, etc., whilst the columns headed x and y give the distances OB, BC, CD, B1, F2, H3, etc., and are obtained by subtracting from any value in the XY columns the corresponding value on the line above. The column headed " Individual chord-length " is obtained by taking the square root of the sum of the x^2 and y^2 columns, whilst the last column is the sum of the individual chord-lengths. Actually, within

the limits of application of the cubic parabola little error would be intro duced by assuming that the subchord-length equals X. The formula (the cubic parabola being $y = kx^3$, where k is a constant $= \dfrac{1}{6RL}$ (R denotes the minimum radius of the transition and L the length of th transition), no difficulty should be met in filling in the above Table.

It is not proposed to give more than this cursory description of settin out the cubic parabola, since its application to road-work is not of suffi ciently general use, but it should be pointed out that exactly the sam methods can be used as are later to be described for the spiral and lemnisi cate. With regard to a suitable length for the cubic parabola, Shortt't formula may be used, as although this is strictly applicable only to th clothoid or true spiral, the error introduced by using it on the cubic para bola is small, provided a polar deflexion-angle of 4 or 5 degrees is no exceeded.

Another formula, an approximation to the cubic parabola, is that use by Mr. Henry Criswell in setting out curves on the Great North road which is :—

Deflexion-angle in minutes $= \dfrac{N^2 L}{10}$, where L denotes the length of th subchord in feet and N the number of the subchord point. The American degree system is used in the application of this formula, and the distance required for a change of 1 degree in curvature is chosen as the length o subchord. This curve begins to become inaccurate after 5 degrees, an hence has little advantage in this respect over the curve previously described.

Another equation, $y = kx^2$, is sometimes suggested for use as a tran sition-curve for horizontal alignment [1], but its use lies chiefly in its appli cation to vertical curves.

The Cubic Spiral.

The cubic parabola is the result of applying two approximations to the formula for the spiral. It may be shown that for the spiral

$$x = \sqrt{2RL}\left(\phi^{\frac{1}{2}} - \frac{\phi^{\frac{5}{2}}}{5\underline{\lfloor 2}} + \frac{\phi^{\frac{9}{2}}}{9\underline{\lfloor 4}} \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \right) \quad (1)$$

and

$$y = \sqrt{2RL}\left(\frac{\phi^{\frac{3}{2}}}{3} - \frac{\phi^{\frac{7}{2}}}{7\underline{\lfloor 3}} + \frac{\phi^{\frac{11}{2}}}{11\underline{\lfloor 5}} \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \right) \quad (2)$$

where R and L are the same as for the cubic parabola and ϕ denotes the deviation-angle.

[1] H. J. Collins and C. A. Hart, " Principles of Road Engineering." London, 1936.

If when deducing these formulas it is assumed that $\sin \phi = \phi$, then

$$y = \sqrt{2RL}\left(\frac{\phi^{\frac{3}{2}}}{3}\right),$$

which gives

$$y = \frac{l^3}{6RL}\left(\text{since } \phi = \frac{l^2}{2RL}\right) \quad . \quad . \quad . \quad . \quad . \quad (3)$$

If in addition it is assumed that $\cos \phi = 1$ or $x = l$, the cubic parabola is obtained. The formula $y = \frac{l^3}{6RL}$ defines what may be called the cubic spiral; it gives a method of setting out a transition-curve which, although still far from the ideal, is a great advance on the two forms already described. In making the necessary calculations for this curve, it is essential to assume that $RL = a$ constant; this approximation may be taken as valid within the limit of application of the curve, which is up to a polar deflexion-angle of about 6 degrees or a total deviation of 36 degrees. This curve is in effect a foreshortening of the spiral formula; hence in determining ϕ it is an approximation to assume that $\phi = \frac{l^2}{2RL}$, but either this or the approximation $\phi = 3\alpha$ is necessary in setting out the curve.

Before going on to the more accurate forms of transition-curves, a suggested Table for making the necessary calculations for the cubic spiral will be given.

TABLE II.

Chord-point.	Sub-chord, l.	Σl	Y	y	y^2	l^2	x	$X = \Sigma x$	$\tan \alpha = \frac{Y}{X}$	α
1.										
2.										
3.										
4.										
5.										

The column headed Y follows from substituting the values of Σl in the formula of the curve, whilst the y-column is obtained by subtracting from the particular value of Y the value of Y on the previous line. The x-column is obtained from the square root of the difference of l^2 and y^2, whilst the X-column is merely the summation of x. For setting out, the columns used would be the second and last, each chainage following on from the last chord-point fixed.

The Clothoid or True Spiral.

The general formula for the spiral is

$$L = m\sqrt{\phi}, \quad \cdots \cdots \cdots \quad (4)$$

where m is a constant $= \sqrt{2RL}$. It may be shown that for the spiral is constant, where r denotes the radius at distance l along the curve. The true spiral is that curve traced out by a car whose steering wheel is being turned at a constant rate (assuming there is no slip angle, for a description of which see p. 345). The spiral has the disadvantage that, unlike the lemniscate, it has no simple relation between ϕ and α and the calculations are accordingly greatly increased.

For the spiral,

$$\tan\alpha = \frac{y}{x} = \frac{\phi}{3} + \frac{\phi^3}{105} + \frac{\phi^5}{5,997} - \frac{\phi^7}{198,700} \quad \cdots \quad (5)$$

Before a transition-curve can be set out, the parts which must be either decided or given are :—

(i) the speed-standard,
(ii) the maximum permissible value of the centrifugal ratio B,
(iii) the maximum permissible value of the rate of gain of centripetal acceleration C,
(iv) the total deviation ϕ required.

The requirements (i) and (ii) immediately fix the minimum value of R, the smallest radius of the curve. In this connexion it may be stated that when an improvement is being carried out and a transition is being inserted to an existing circular curve, it is advantageous, if at all possible, to ignore the existing curve and proceed from first principles, as if two straights with a certain deviation have to be joined by a transition curve. It should be kept in mind that the curve requiring the least set-back from the intersection-point (that is, the distance AX in *Fig. 2*) is that which develops the maximum possible value of B at the designed speed, being the curve in which no foreshortening of the transition is caused by the premature insertion of a circular arc. If the space available still prevents the ideal treatment visualized, then either the permissible value of C has to be increased or—what is, in effect, the same thing—the speed-standard of the curve must be reduced. On the other hand, in cases where there is plenty of room available, if the value of ϕ is large it is an advantage to keep the minimum radius R as large as possible and, hence, the value of B low. In this way, although a circular arc has to be inserted, an actual shortening of the roadway will result owing to the decrease in length of the transition and approach-straight. This is made clear by *Fig. 2*.
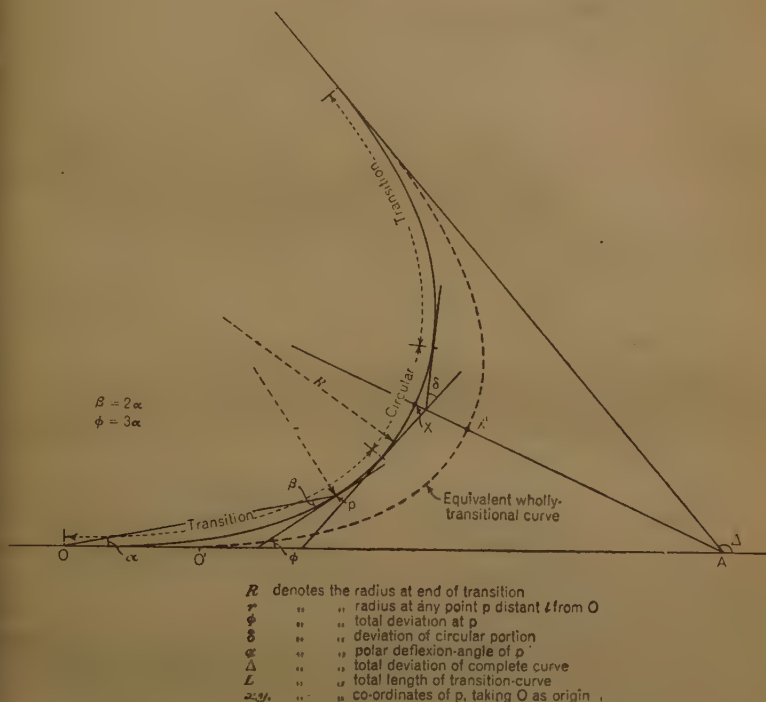
Items (i) and (iii) automatically fix the " scale " of the transition-curve and when coupled with requirement (ii) fix L, the total theoretical length

of the transition. The meaning of the term " scale " as applied to transi-
tion-curves is illustrated graphically in *Fig. 3* (p. 336).

Item (iv), together with (i) and (ii), enables the following questions to
be settled :—

> (a) Whether the maximum value of B or the minimum permissible
>   value of R can be reached and, if not, the extent to which the
>   possible values of B and R fall short.
> (b) Whether a circular arc is required, and, if so, of what length.

Fig. 2.

R denotes the radius at end of transition
r „   „ radius at any point p distant l from O
ϕ „   „ total deviation at p
δ „   „ deviation of circular portion
α „   „ polar deflexion-angle of p
Δ „   „ total deviation of complete curve
L „   „ total length of transition-curve
$x, y,$ „   „ co-ordinates of p, taking O as origin

Before proceeding further it is necessary to study the connexions
between the speed-standard, B, and the minimum radius, R, and between
the speed-standard, C, and the minimum length, L. The first follows from
the ordinary dynamic law of centrifugal force, and may be expressed as

$$V^2 = 14 \cdot 969 BR \text{ (where } V \text{ denotes the vehicle-speed in miles per hour)} \quad (6)$$

The reasoning for the second relation was given by Mr. W. H. Shortt,
and is as follows :—

The time taken to traverse the transition is $\dfrac{L}{v}$ seconds, and the final

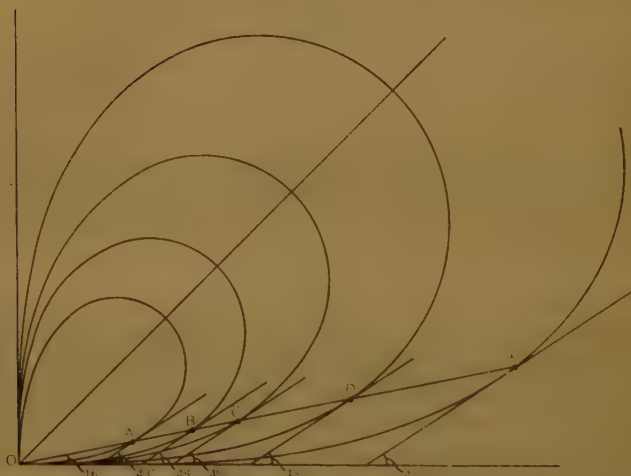centripetal acceleration $= \dfrac{v^2}{R}$. Hence the rate of gain of centripetal

acceleration $C = \dfrac{v^3}{LR}$, so that

$$L = \frac{v^3}{CR} = \frac{3 \cdot 155 \, V^3}{CR} \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Alternatively, L may be expressed entirely in terms of V and of the standards C and B thus :—

$$L = \frac{3 \cdot 155 \, V^3}{C \times V^2} \times 14 \cdot 969 \, B = \frac{47 \cdot 227 \, VB}{C} \quad . \quad . \quad . \quad . \quad (8)$$

Fig. 3.

THE CHORDS OA, OB, OC, OD, AND OE ALL SUBTEND THE SAME POLAR DEFLEXION-ANGLE, BUT HAVE LENGTHS PROPORTIONAL TO THE SCALE OF THE CURVE

Further, L may be expressed entirely in terms of R and of C and B, thus :—

$$L = 3 \cdot 155 \frac{(14 \cdot 969 \, BR)^{\frac{3}{2}}}{CR} = \frac{182 \cdot 72 \sqrt{RB^3}}{C} \quad . \quad . \quad . \quad . \quad (9)$$

The stage has now been reached when ϕ, R, and L are all fixed at their preliminary values. The next step is to determine whether a circular arc is necessary or desirable, and hence whether the values of R and L will have to be modified. It is assumed that L has been calculated by Shortt's method. This then is the theoretically-required value of L for the speed value attached to the curve. It now remains to see if this value of L gives more or less than the given value of ϕ. In the former case the transition

will have to be cut short at the point where the required deviation has been attained, the minimum value of R thus reached will be greater than the minimum allowable value of R, and a circular arc will not be necessary. In the latter case the transition will be prolonged to its full theoretical length with the consequent attainment of the limiting values of R and B, at which point the curve must continue as a circular arc until half the given total deviation is taken up, whereupon the reverse half of the curve, which is a mirror image of the first, is entered.

The question of whether a circular arc is in any case desirable is a matter to be decided by local circumstances. If the bend is free from obstacles, then it may be considered expedient to adopt a larger minimum radius than that theoretically permissible. In this case there will be a decrease in the value of B, but this may not be increased by greater speed, as the whole basis of the design is the adoption of a fixed speed-value for all curves on the particular road ; that speed at once fixes the scale of the transition, which will, in this case, be the limiting factor.

To determine whether a circular arc is theoretically necessary, the value of L, determined by formula (7), (8), or (9), should be substituted in the general equation of the spiral, namely, $L = m\sqrt{\phi}$. If by an application of the foregoing reasoning it is necessary to shorten the spiral, then it will be required to determine to what length. If the original length of transition consumes too great a deviation and it is desired to have the curve transitional throughout, then the resultant value of L may be obtained by substituting half the total required deviation in $L = m\sqrt{\phi}$. If, on the other hand, it is decided to increase the theoretical value of R, then the new value of L may be determined from the relation $lr = $ a constant. It should be noted that this constant lr is a value determined by the scale of the transition and may be obtained from the preliminary values of L and R or from formula (7) ; thus,

$$LR = \frac{3 \cdot 155 V^3}{C}$$ (where V denotes the speed in miles per hour associated

with the given scale).

The value of L having been determined finally, the subchord-length l may then be fixed at a suitable figure. All that it is then necessary to do is to calculate the values of α for the various chord-points. The formula for doing this may be obtained by substituting for ϕ from equation (4) in equation (5), whence

$$\tan \alpha = \frac{l^2}{3m^2} + \frac{l^6}{105m^6} + \frac{l^{10}}{5997m^{10}} - \frac{l^{14}}{198700m^{14}} \quad . \quad . \quad (10)$$

The application of this formula is undoubtedly somewhat tedious, but it will be found that if the tabulated method of procedure, to be described, is followed then the time taken and the chances of mistake are greatly reduced.

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The method that has just been outlined makes one approximation; namely, that Σl equals the true length along the curve. By reducing the subchord-length l this error may be made as small as necessary; for all practical purposes it is safe to make $l = 50$ feet, and with discretion this may be increased.

Moving of Theodolite.—It is now proposed to deal with the case where it is necessary to move the theodolite to an intermediate point for setting out the latter part of the curve; indeed, it might well be considered that this is a desirable procedure on any curve, as the lengths of the sights are thereby shortened and the number of subchords for which α has to be calculated from formula (10) is decreased. The law of the osculating circle is fully discussed in other literature [1], but for the sake of completeness it will be described briefly here, as an application of its principles is almost essential if the position of the theodolite is to be moved.

Fig. 4.
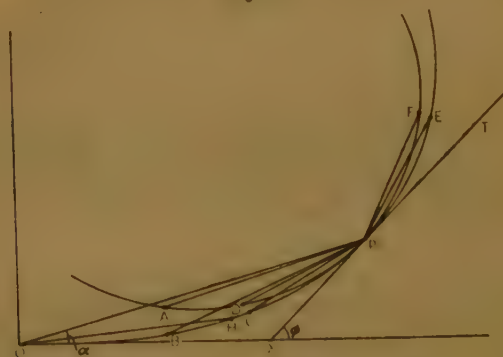
In *Fig. 4*, if p is any intermediate point on the spiral OBCpF, and PX is the tangent to the spiral and circle ADE touching at p, then :—

$$X\hat{p}C = X\hat{p}D - D\hat{p}C,$$
$$X\hat{p}B = X\hat{p}A - A\hat{p}B,$$

also

$$F\hat{p}T = E\hat{p}T + F\hat{p}E.$$

The circle ADpE is the osculating circle; it has a radius equal to that of the spiral at point p, and hence $X\hat{p}D$, $X\hat{p}A$, $E\hat{p}T$, can all readily be calculated. The law of the osculating circle states that $D\hat{p}C$, $A\hat{p}B$, and $F\hat{p}E$, where $Dp = pC$, $Ap = pB$, $Fp = pE$, are respectively equal to the polar deflexion-angles subtended from O by chords of length equal to Dp, Ap, Fp. It is therefore simple to calculate the deflexion-angles from the tangent XT at p. In general terms the deflexion-angles will be $na_c - a_n$ for the backward direction, and $na_c + a_n$ for the forward direction.

[1] Footnote (2), p. 327.

where a_c is the osculating-circle unit deflexion and a_n the spiral deflexion for the nth chord from the change point.

In addition to calculating these deflexions, it is necessary to know how to locate the tangent at p, for which purpose a knowledge of the back angle $O\hat{p}X$ is necessary. The angle α or $p\hat{O}X$ is already known from the forward setting out from O. By assuming the length of the transition $L = \Sigma l$, the value of ϕ may be calculated from $L = m\sqrt{\phi}$. Hence the back angle $= \phi - \alpha$ can be calculated. A useful check on the calculations can be obtained by testing to see if $na_c - a_n = \phi - \alpha$, n here being taken as the number of chord-lengths to the point p. The two sides of this equation will not agree exactly, owing to the assumption made that $\Sigma l = L$, but the difference will be negligible.

In applying the law of the osculating circle a further assumption is made, namely, for example, that the chord pF is of the same length as the chord OH, where H may be, say, the third point set out from O by measuring three smaller chords round the curve, and F the third point located from p by measuring three small chords (equal to those measured from O) from p, but the effect of this assumption is usually not serious. Also, it is assumed that Fp and Ep are equal, where F and E are located by measuring three smaller chords, equal in each case, respectively round the spiral and round the osculating circle.

Determination of Shift and Total Tangent-Distance.—The shift and the total tangent-distance may be obtained as follows :—

X and Y should first be calculated from the formulas :—

$$X = m\sqrt{\phi}\left(1 - \frac{\phi^2}{5\lfloor 2} + \frac{\phi^4}{9\lfloor 4} - \frac{\phi^6}{13\lfloor 16} + \frac{\phi^8}{17\lfloor 8} - \frac{\phi^{10}}{21\lfloor 10} + \cdots\right)$$

and

$$Y = m\sqrt{\phi}\left(\frac{\phi}{3} - \frac{\phi^3}{7\lfloor 3} + \frac{\phi^5}{11\lfloor 5} - \frac{\phi^7}{15\lfloor 7} + \frac{\phi^9}{19\lfloor 9} - \cdots\right)$$

Several terms have been given, but more than the first three will rarely be necessary.

From *Fig. 5* (p. 340), $pD = R\sin\phi = FB$.

Hence the tangent-distance to the beginning of the transition, or the distance OI (where I is the intersection point)

$$= (R + S)\tan\frac{\Delta}{2} + (X - R\sin\phi), \quad \ldots \quad (11)$$

where Δ denotes the total deviation of the curve and S denotes the shift.

The shift S may be determined as follows :—

$$CD = R\cos\phi,$$
$$pB + CD = CF,$$
$$\text{also } S = CF - R$$
$$= (Y + R\cos\phi) - R.$$

Hence $\qquad\qquad S = Y - R(1 - \cos\phi) \quad . \quad \ldots \quad . \quad (12)$

The Lemniscate.

The formulas of the lemniscate are :—

$$\rho = 3R \sin 2\alpha, \quad \text{.} \quad \text{.} \quad \text{.} \quad \text{.} \quad \text{.} \quad \text{.} \quad \text{.} \quad \text{.} \quad (13)$$

and

$$\rho = k\sqrt{\sin 2\alpha}, \quad \text{.} \quad \text{.} \quad \text{.} \quad \text{.} \quad \text{.} \quad \text{.} \quad \text{.} \quad \text{.} \quad (14)$$

where ρ denotes the polar chord and $k =$ a constant $= \sqrt{3\rho R}$

Fig. 5.
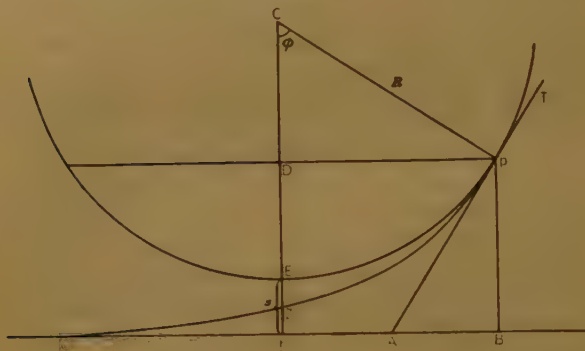
In addition, a very useful formula, of which use will be made, is the speed–chord formula evolved by Professor Royal-Dawson. He has shown [1] that under his unit-chord system for a polar deflexion-angle of 16 minutes, when L has its unit value, $R = 35\cdot 81D$, where D denotes the length of unit chord.

Hence, by substituting in formula (7),

$$D = \frac{3\cdot 155 V^3,}{35\cdot 81 DC}$$

or

$$11\cdot 4CD^2 = V^3 \quad \text{.} \quad \text{.} \quad \text{.} \quad \text{.} \quad \text{.} \quad \text{.} \quad (15)$$

where C denotes the allowable rate of gain of centripetal acceleration and V the speed in miles per hour. Professor Royal-Dawson substitutes $C = 1$ foot per second per second per second, which he adopts as a permanent value.

The lemniscate, unlike the spiral, has the valuable property that $\phi = 3\alpha$ exactly, for all values. The lemniscate, on the other hand, has a mathematical complication arising from the fact that ρr is a constant, instead of lr being constant, as in the spiral. This means that if at the start of the curve $C = 1$ foot per second per second per second, at the end of the curve its value will be slightly below that figure. It further means that Shortt's

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[1] " Road-Curves," p. 204. London, 1936.

value for the length of transition is no longer strictly valid; indeed, the exact formula for the length of the lemniscate is a complicated series which at the higher values of ϕ does not converge at all rapidly. In actual practice an approximation may be made whereby the value of L can be determined with sufficient accuracy for all normal purposes. This approximation is the same as was used for the spiral, namely that $\Sigma l = L$, and does in fact hold for very wide deflexion-angles. If l is made equal to one-quarter of the chord-length it may in most cases be applied for polar deflexion-angles as high as 45 degrees, or in other words for a complete loop of the lemniscate. It should further be realized that, with the method of setting out described in this Paper, this assumption does not affect the accuracy with which the two halves of the transition meet; a true lemniscate is in fact set out, the only error being in the recorded length. The constant $k = \sqrt{3\rho R}$ is the maximum polar ray or polar axis, and may be determined in terms of unit chords. The unit that will be adopted is that used by Professor Royal-Dawson, namely, the length of polar ray whose polar deflexion is 16 minutes. Under this system $k = 10\cdot3648$ units; hence

$$\rho = 10\cdot3648 D \sqrt{\sin 2\alpha} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

It is therefore easy to determine D from the speed–chord formula; ρ may then be found for various values of α. The curve may, if desired, be set out by polar rays all measured from the origin, but this is a rather cumbersome method involving the chaining of an unnecessary number of long distances, and leaving the easy mathematical determination of L still unsolved.

If a lemniscate is to be set out the quantities given will be ϕ and the speed-standard, the values of B and C being decided at the engineer's discretion. From the speed-standard D may at once be determined, and by an application of B, the minimum theoretical radius may be determined, from formula (6). If the curve is to be wholly transitional the maximum value of $\alpha = \dfrac{\phi}{6}$. This value may then be substituted in equation (16) and the corresponding value of ρ found. On then substituting ρ in the equation $\sqrt{3\rho R} = k = 10\cdot3648$, or ρ and α in equation (13), R may be found. If this value of R is smaller than the minimum allowable value fixed by B, then the curve cannot be wholly transitional and a circular arc must be introduced. If, on the other hand it is larger, then the curve can be transitional throughout.

If the engineer should consider it preferable, or if he is obliged by natural features, to fix a value for R greater than the minimum theoretical figure, then R should be substituted in $\sqrt{3\rho R} = 10\cdot3648$, and the corresponding value of ρ found. It should be remembered that the formula $\sqrt{3\rho R} = 10\cdot3648$ is in unit chords, and consequently the right-hand side must be multiplied by D if R is inserted as its actual value in feet. The value of ρ thus obtained will then also be in feet.

At this stage then it might be advisable to write the above formula
in the form

$$\rho R = 35 \cdot 8097 D^2, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

where ρ and R are to be given in the same units as D.

When the value of ρ corresponding to the given value of R has been
obtained, these can be substituted in equation (13) and the value of α and ϕ
at the end of the transition thus found. From equation (16) corresponding
values of α and ρ may then be found up to the final value of α. In making
these calculations it should be remembered that the arc-length varies
approximately as the square of the angle α; hence, in order to keep the
values of l as finally calculated approximately equal, the values of α sub-
stituted in the formula should ascend according to the square law so that
$\alpha_n = \alpha_1 n^2$. If a small radius is reached, then the progressive intervals in
the values of α should be reduced slightly in order to decrease the subchord
lengths as the radius of curve set out decreases. As the method of setting-

Fig. 6.
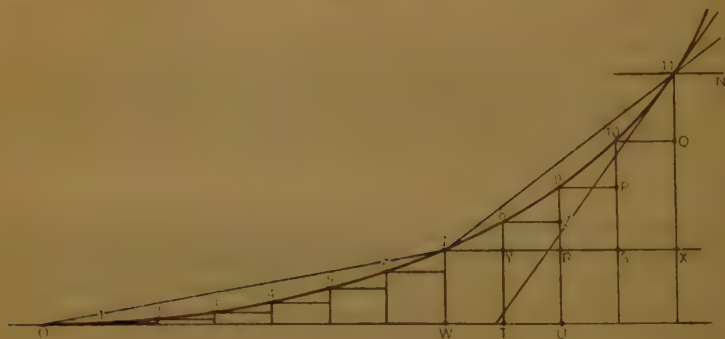
out by polar rays is objectionable, it is desirable to obtain the chord-
lengths from the values of ρ and α. This may be done in a similar way to
that adopted for the cubic parabola, and a tabular arrangement is therefore
submitted in the example on the lemniscate (Appendix II, p. 358).

Examples are given in Appendixes I and II to illustrate the foregoing
methods of setting out spiral and lemniscate transition-curves.

One of the great disadvantages of the lemniscate is the fact that the
law of the osculating circle does not hold exactly. If, however, the tabular
method set forth in this Paper be adopted, the position of the theodolite
may be changed any number of times without introducing much compli-
cation, the calculations being very similar to those of Appendix II. Thus
if in *Fig. 6* it is desired to move the theodolite to station-point 7 it will be
necessary first to locate the line 7 X and then to calculate α_1, the deflexion-
angle from the line 7 X to the subsequent station-points. The line 7 X may
be located by setting up the theodolite over station-point 7, sighting on to

point O, and then turning it through an angle equal to $7OW$; if it is now transited the theodolite will point in the direction $7X$. The deflexion-angle from station-point 7 to station-point 8 is $\tan^{-1}\left(\dfrac{8Y}{Y7}\right)$.

Now $8Y = 8T - 7W$, and $Y7 = TO - WO$. Similarly, the deflexion-angle from station-point 7 to station-point 9 is $\tan^{-1}\left(\dfrac{9R}{R7}\right)$, where $9R = 9U - 7W$ and $R7 = UO - WO$. The calculations can conveniently be made by adding four columns to Table D of Appendix II, namely, column 12, x_{11}; column 13, y_{11}; column 14, $\dfrac{y_{11}}{x_{11}} = \tan \alpha_1$; and column 15, α_{11}. The distances $Y7$ and $R7$, etc., and $8Y$ and $9R$, etc., will appear in columns 12 and 13 respectively and are obtained in each case by subtracting the value of y and x against station-point 7, in columns 6 and 5, from the successive values of y and x against each respective chord-point. The subtraction of the logarithms of y_{11} and x_{11} is then entered in column 14, and α_1 is entered in column 15.

If a circular curve has to be set out from station-point 11, the tangent $11T$ at this point may be located by sighting the theodolite on to station-point 7, transiting, and turning counter-clockwise through an angle equal to the deviation ϕ, for station-point 11, minus the deflexion-angle from line $7X$ of station-point 11.

## CASES WHERE TRANSITION-CURVES MAY BE OMITTED.

It is evident that the lower the speed-standard or the greater the minimum radius, the smaller will be the shift of the circular curve necessary for the insertion of the transition. Below a certain value of shift it may well be considered that the transition is unnecessary; this is indeed the attitude of Mr. Criswell, but on the other hand it is the practice in the state of Oregon [1] to insert transition-curves at all changes in direction irrespective of the shift, the reason being that the length of transition is an indication of the length of run-off required in reducing the superelevation. It is, however, a simple matter to calculate the length of transition from formula (7), (8), or (9); alternatively, when dealing with such low values of superelevation without the provision of transition, there can be no logical objection to effecting the necessary run-off or application of superelevation by one of the gradient methods, to which reference has already been made. Actually a rate of gain of centrifugal ratio of $\dfrac{C}{32}$ per second

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[1] R. H. Baldock, "Highway Design for Speeds Up to 100 Miles per Hour." *Engineering News-Record*, vol. 114 (1935), p. 732.

is equivalent to a rate of gain of centripetal acceleration of C feet per second per second per second. Thus if the "0·4 rule" is adopted a rate of application of superelevation of $0·4 \times \dfrac{C}{32}$ will give the correct basis of design.

Fig. 7 gives the radius of curve above which it is unnecessary to apply transition for a given speed-standard. Three values of shift are given the first that taken by Mr. Criswell (0·20 foot) after which a transition-curve becomes necessary, and the last a higher value of shift (0·5 foot)

Fig. 7.
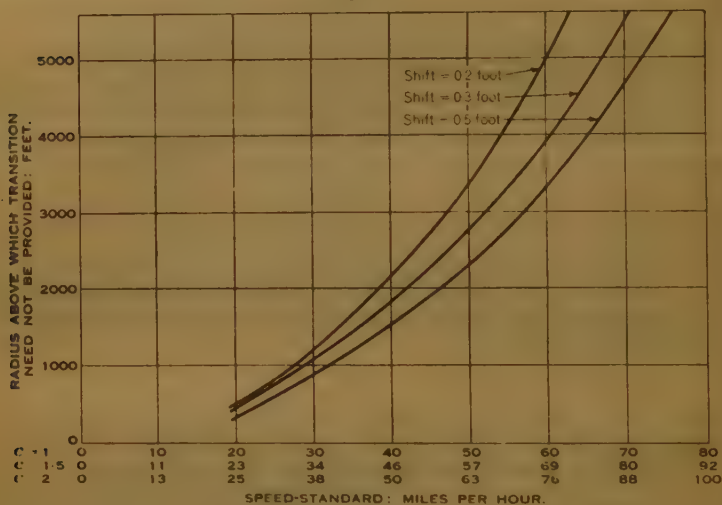
which owing to the undefined path of a car might well be taken as the limit for wide roads. Intermediate values may be obtained by interpolation, according to the requirements of the engineer concerned.

SUPERELEVATION.

Recognition of the importance of superelevation came before general consideration was given to the transition-curve as a necessary part of highway-engineering. Much has been written on the theory and practice of superelevation, but the whole question is still largely controversial. One evident point is that the superelevation must not be too slack to prevent adequate drainage, so that the minimum superelevation is limited to about 1 in 40 for existing road-surfaces when the curve is not on a grade, but the upper limit of superelevation is still a matter of opinion.

According to one rule, the superelevation is made 0·4 of the centrifugal

ratio at any part of the curve, which gives a maximum sideways slope of 1 in 10 if the transition reaches its limiting minimum radius at a value of B = 0·25 ; this slope, however, seems to be higher than that adopted in many cases, and indeed a cross slope of 1 in 14 is about the maximum that can be considered safe for horse-drawn traffic. On the other hand, on new roads of the type of the German motor-roads, which it is to be hoped will eventually be adopted in England, there seems to be no reason why, on the sharp curves which have to be provided at fly-over junctions, a rate of superelevation of as high as 1 in 7·5 should not be used, provided that these connecting roads have single-way traffic, so that half the road-width may be laid to a more gradual cross-fall.

In deciding the maximum superelevation to be allowed there are two further considerations which must not be overlooked when the curve is situated on a hill : Firstly, when a car is accelerating, or when there is an increase in driving force due to its ascending a hill, an appreciable component of the driving force opposes centrifugal action, but the reverse is the case when the car is braking on a descending grade. This fact is further aggravated by the reduced sideways force available when the brakes are being applied (it has been shown by Professor R. A. Moyer [1] that the total available coefficient of friction is the resultant of the sideways coefficient and the braking force coefficient obtaining simultaneously) even though the wheels are not skidding. Secondly, when a car is ascending a grade on a curve, the outer back and front wheels will be further up grade than the inner wheels, thus increasing the effective superelevation, and when the car is descending a grade, the outer wheels will be further down grade than the inner wheels, thus decreasing the effective superelevation. This phenomenon is due to the slip angle assumed by the back wheels, in order to generate the necessary sideways force, and occurs to any pronounced degree only when the car is taking the curve in excess of the speed where superelevation exactly neutralizes centrifugal force (*Figs. 8*, p. 346). The slip angle is the angle between the normal tangential running position of the wheel and the position it assumes when called upon to develop sideways force. In this connexion it should be remembered that at low speeds the rear wheels will track inside the front wheels, and at speeds above that for which the curve is superelevated, the rear wheels will track outside the front wheels. This is determined by the slip angle required for developing sideways force in the correct direction for the particular case.

The second consideration in determining a suitable value of superelevation occurs only when the curve is on a very steep grade, but as that is sometimes the case in mountainous districts, it is worth investigating the extent of its effect. Figures provided by Professor Moyer show that on a curve of 600 feet radius the side slip of the back wheels (the amount by which they track outside the front wheels) may be as much as 10 inches for

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[1] Footnote (5), p. 329.

a car travelling at 50 to 60 miles per hour. Speaking in terms of slip angles, Professor Moyer's diagrams show that a slip angle of quite 4 degrees should be provided for. The wheel-track of the car does not affect the problem ; it will be taken as 5 feet for purposes of calculation. On this basis the amount by which the outside wheels ascend or descend the grade

Figs. 8.

CAR TRAVELLING AT HIGH SPEED
OR ACCELERATING.

CAR TRAVELLING AT SPEED FOR WHICH
CURVE IS SUPER-ELEVATED.

C denotes the total centrifugal force
C_r ,,   ,,   centrifugal force on rear wheels
S_r, S_f ,,   ,,   sideways force on rear and front wheels
F_r, F_f ,,   ,,   resultant frictional force on rear and front wheels
D ,,   ,,   total driving force
D_a ,,   ,,   radial component of driving force
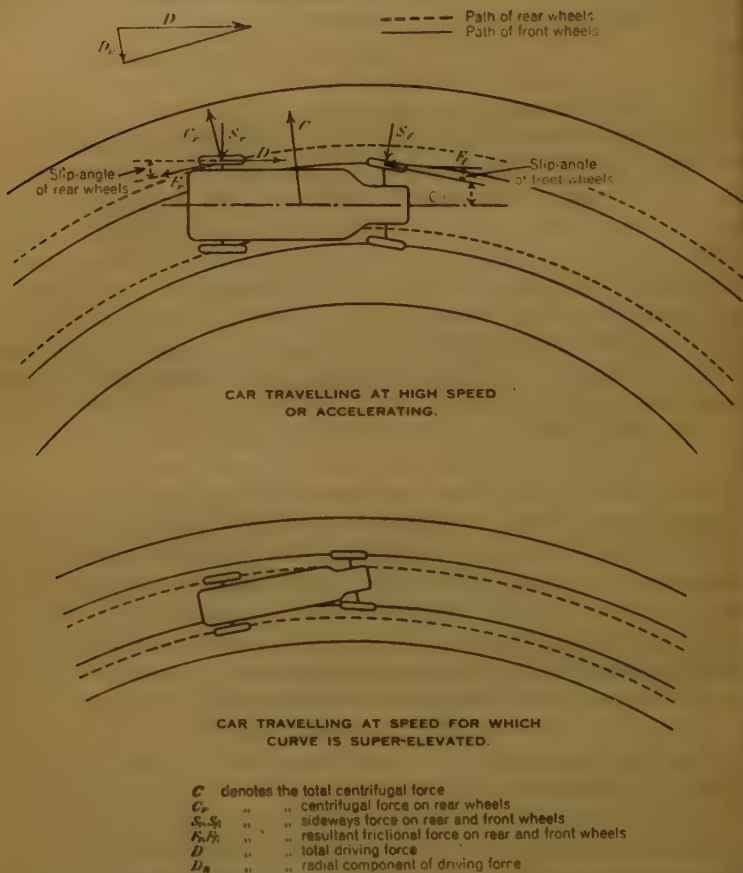
is $5 \sin 4° = 0.35$ foot. Assuming a gradient of 1 in 6, this means an increase or decrease in superelevation of 0·06 foot in 5 feet, or 1 in 83. Considering that an extreme case was taken (but nevertheless one which may occur in mountain roads) this is not a very large factor, but to this must be added the effect of the centripetal component of the braking force of a car descending the grade. Thus if the gradient is 1 in 6, a braking

coefficient of $\frac{1}{6}$, or say 0·25, to allow for a margin to reduce excess speed, is needed.

The centripetal component caused by this will be

0·25W sin 4° (where W denotes the weight of vehicle) = 0·018W (see *Figs. 8*).

In other words, an increase in superelevation of 1 in 56 is required. Thus, in order to maintain balanced dynamic conditions, a total increase in superelevation of 1 in 34 may in an extreme case have to be provided, when a car is descending the grade. As under slippery conditions an increase of this amount in the superelevation would make it difficult for ascending cars, the necessity will be seen of paying attention to the " compensation " of curves by reducing gradients where such mountain conditions arise. If the gradient cannot conveniently be reduced, then it is essential that the ultimate value of B should be made as low as possible.

LOCATION OF TRANSITION-CURVES.

Before a transition-curve can be calculated or set out its basic properties have to be determined. In the case of entirely new motorways, it is desirable that one speed-standard should be maintained for all curves. For this reason the values of B and C should be established and should be maintained on every curve, if at all possible. If the values of B and C have to be increased for any individual curve, there will be a danger of the motorist misjudging his turn through an attempt to maintain the general average for which the road is designed. In this connexion it would be helpful in the prevention of accidents if notices indicating the safe speed on all main roads were provided ; any departure from this safe speed, or any increase in the value of C or B, should be brought to the attention of motorists in some suitable manner.

After the desired speed-standard for a new road has been fixed, or, in the case of a curve improvement, after the speed-standard has been fixed to fit in with the existing curves, it is possible to locate the new curves on a plan. This may be done by means of the curve-plotters developed by Professor Royal-Dawson, or by means of the transition-curve locator described by Mr. E. W. W. Richards. Suitable curve-plotters may be chosen to suit the speed-standard and scale of the drawing. The plotters or locator may then be adjusted until the centre-line can be drawn in to fit the local conditions. It is then possible to determine approximately the deviation of the transition, and hence to calculate preliminary values for the remaining properties of the curve, before taking more accurate theodolite measurements.

At this stage it must be decided whether the curve is to be wholly transitional, or is to include a circular portion. This will be dictated largely

by local conditions or obstructions, provided always that the limitations imposed by the values of B and C are met. In the case of a road-improvement it may be necessary to approach as near to the intersection-point as possible, in which case the full value of B will be developed, but when the engineer has a free hand there is no reason why B should not be kept very small by the insertion of a curve having as large a radius as possible. In this way the necessary superelevation may be kept small, which is a very great advantage under ice conditions, and is an ideal to be aimed at under all conditions. In this connexion it may be stated that, according to Professor Moyer, a value of B of 0·3 in excess of that taken up by super-elevation is the limit for the comfort and physical powers of the driver.

WIDENING ROADS ON CURVES.

At this stage attention should also be given to the advantages accruing from a widening of the road on curves in order to allow for longer sight-distances and for the increased space taken by a vehicle on a curve. For radii above about 700 feet the theoretical widening does not exceed 1 foot, and thus it may be taken that only curves of smaller radius than that need to be so treated. The theoretical requirements of widening on curves are considered by some to be limited by the fact that excessive widening will encourage some drivers to make three traffic-lanes in a nominal two-lane road, thus defeating the objects of the engineer in widening the road at this point. If this limitation is accepted the widening should not exceed about 3 feet, it being assumed that a band of unobstructed ground is left on the inner edge of the road to provide for adequate sight-distances. The method where each side of the road is set back from the centre-line by a distance equal to $\frac{1}{2}W \operatorname{cosec}\left(90 - \frac{\triangle}{2}\right)$, measured everywhere parallel to the line joining the intersection-point with the centre of the curve, has the advantage that both edges of the road are maintained as true lemniscates or spirals of identical scale, but, if the above consideration is adopted its application will be limited to the deviation-angle at which $W \operatorname{cosec}\left(90 - \frac{\triangle}{2}\right) - W$ becomes greater than 3 feet, that is,

$$W\left\{\operatorname{cosec}\left(90 - \frac{\triangle}{2}\right) - 1\right\} \ngtr 3 \text{ feet};$$

$$\text{that is, } W\left(\sec\frac{\triangle}{2} - 1\right) > 3 \text{ feet.}$$

Taking W as 20 feet this limits $\triangle$ to 59 degrees 12 minutes, or say 60 degrees. This method of widening will therefore apply to most road-curves. If the

engineer desires to limit the widening to 3 feet, then for values of $\triangle$ above 60 degrees the only method is to lay the inner edge out to a slightly smaller scale than the outer edge, unless some arbitrary or empirical method is adopted, a practice which is not to be advised as the properties of the curves would thereby be altered. Of course, for sharp curves of large deviation such as may occur on mountain roads, this limit of 3 feet for the widening is rather small.

Conclusion.

A realization of the potentialities of the correct transitioning of curves may lead to a revolution in highway-alignment. Far from it being desirable to take the route giving the straightest road, curves may be inserted purposely without in any way affecting the speed-standard or safety of the road. Where a twin carriage-way with a dividing hedge down the centre to prevent headlight glare cannot be provided, attention should be directed to the possibility of inserting regular large-radius curves set out to a constant speed-standard. In this way much would be done to lessen the danger of headlight-glare, which is such a fruitful source of road-fatalities, and incidentally the objections of camber would be overcome by the provision of correct superelevation.

There is still much scope for research in the matter of transition-curves. The permissible value of C is still largely controversial, and the necessity for its establishment at a fixed value, to be maintained wherever topographical conditions permit, is a matter badly in need of official attention in Great Britain. Preliminary tests by the Author point to the possibility of a variation in the value of sideways force-coefficient attainable if the sideways force is applied quickly. This is a point worthy of the closest attention, as in order to obtain a safety-factor in the value of B under bad conditions it may be deemed advisable to depart from the value of $C = 1$ foot per second per second per second, a figure chosen largely from considerations of comfort. The effect of weight and speed of vehicles on the value of C are also factors which are not yet fully understood, whilst there is still need for investigation of compensation for grade on curves.

The values of B and C taken in the examples are not meant to indicate any recommendation on the part of the Author. There are circumstances where physical features and other considerations preclude a value of $C = 1$ foot per second per second per second, or $B = 0.25$, and there are engineers who consider that that value of C places too high a standard on comfort. For these reasons the formulas in this Paper have been modified to the extent of keeping B and C as symbols in the final form, and it was to illustrate the flexibility thereby introduced that values other than 1 foot per second per second per second and 0.25 were taken in two of the

examples. In the meantime, it is left to the engineer to decide what standards he will adopt, with a strong plea that wherever possible they should be kept at a regular value and not fixed indiscriminately.

The Paper is accompanied by eleven sheets of drawings, from some of which the Figures in the text have been prepared, by seven photographs, and by four worked examples of the application of the formulas, two of which are arranged as Appendixes I and II.

APPENDIX I.

Suppose that a road-curve has to be set out round a shoulder of rock, and that in order to minimize the expense of cutting this rock it is desired to approach as near to the intersection-point as possible. (Another circumstance under which it may be necessary to approach as near as possible to the intersection-point is when church property would have otherwise to be taken.) It is for this reason considered necessary by the engineer to increase the value of C to 1·15 feet per second per second per second and the value of B to 0·3. The speed-standard is to be 40 miles per hour, and the total deviation to be 103° 39′ 00″. The type of curve used is to be the spiral.

The minimum theoretical radius is given by formula (6) (p. 335),

$$V^2 = 14\cdot969 \times BR,$$

where

$$R = \frac{40 \times 40}{14\cdot969 \times 0\cdot3}$$

$$= 356\cdot292 \text{ feet.}$$

The preliminary value for L is given by formula (8) (p. 336),

$$L = \frac{47\cdot227 VB}{C}$$

$$= \frac{47\cdot227 \times 40 \times 0\cdot3}{1\cdot15}$$

$$= 492\cdot804 \text{ feet.}$$

The value of ϕ taken up by each half of this transition is given by formula (4) (p. 334),

$$L = m\sqrt{\phi};$$

thus

$$(492\cdot804)^2 = (2 \times 492\cdot804 \times 356\cdot292)\phi,$$

whence

$$\phi = 0\cdot6915733$$

$$= 39° 37′ 27″$$

This gives a total deviation for the two transition-curves of 79° 14′ 54″. The deviation between the straights must be 103° 39′ 00″, so that a circular section will be necessary. The present case is obviously one in which no increase in the minimum radius attained is desirable, since it is required to keep the apex distance AX (*Fig.* 2, p. 335) as short as possible.

The preliminary values of L and of R will then hold good, and hence the length of subchord l may now be decided. A suitable value for l may be taken as 50 feet for the first nine chord points and 42·80 feet for the last chord point.

It will be assumed in the present example that, because of an obstruction to the line of sight, it is necessary to move the theodolite up to the station-point distant 300 feet from the origin of the curve. The radius at this point may be calculated from the relation $LR = \text{constant}$;

hence

$$300R = 492\cdot804 \times 356\cdot292$$

so that

$$R = 585\cdot27 \text{ feet}$$

The angle subtended by a 50-foot chord at the centre of this circle

$$= 2 \sin^{-1} \frac{25}{585\cdot27}$$

$$= 2 \sin^{-1} 0\cdot0427151$$

$$= 2 \times (2° 26′ 53″)$$

Hence the deflexion angle for a 50-foot chord = 2° 26′ 53″. Similarly the deflexion for a 42·80 foot chord

$$= \sin^{-1} \frac{21·40}{585·27}$$
$$= 2° 5′ 44″.$$

The values of the polar deflexion-angle α may now be calculated from formula (10) (p. 337) :—

$$m = \sqrt{2\overline{RL}} = \sqrt{2 \times 356·292 \times 492·804}.$$

$$\log 2 = 0·3010300$$
$$\log 356·293 = 2·5518060$$
$$\log 492·804 = 2·6926742$$

$$2 | \overline{5·5455102}$$

Hence
$$\log m = 2·7727551.$$
$$\log 3 = 0·477121$$
$$\log 105 = 2·021189$$
$$\log 5997 = 3·777934$$
$$\log 198700 = 5·298198$$

2 log m	6 log m	10 log m	14 log m
5·545510	16·636531	27·727551	38·818571
0·477121	2·021189	3·777934	5·298198
6·022631	18·657720	31·505485	44·116769
= log $3m^2$	= log $105m^6$	= log $5997m^{10}$	= log $198700m^{14}$

These are the values which in Table A must be subtracted from the logarithms of l^2, l^6, l^{10}, and l^{14}.

The system of tabulation employed in Table A is really self-explanatory. The number of terms which have to be taken in the series can easily be seen from the index of the logarithm. Thus, if the Table is being worked to six decimal places all terms with an index of $\overline{6}$ and some with an index of $\overline{7}$ must be considered. Column 1 may be regarded as the length of the transition-curve to the particular point being set out. The figures that should first be entered up in the 3rd, 4th, 5th, and 6th columns represent the logarithms of the numerators of the different terms of the series. Under each of these should be put the respective logarithms of the denominators of the terms in the series. The subtraction may then easily be made and the antilogarithms of the values so obtained entered up for addition in column 7.

Table A will permit the curve to be set out as far as $l = 300$ feet. When this point has been located the theodolite must be set up over it and sighted back to the origin O (*Fig. 9*). If it is then turned through an angle $\phi - \alpha = \text{O}\hat{p}\text{A}$ and transited, it will be pointing along the line Ap which is tangent to the spiral at that point; ϕ is given by

$$\phi = \frac{l^2}{m^2}.$$

From previous calculations
$$\log (l^2) = 4·954242$$
$$\log (m^2) = 5·545510$$

Hence
$$\log \phi = \overline{1}·408732$$

TABLE A.

1	2	3	4	5	6	7	8
Σl	$\log l$	$2 \log l$	$6 \log l$	$10 \log l$	$14 \log l$	$\tan \alpha$	α
50	1·698970	3·397940 6·022631 ——— $\overline{3}$·375309	10·193820 18·657720 ——— $\overline{9}$·536100			0·002373	0° 8′ 9″
100	2·00	4·00 6·022631 ——— $\overline{3}$·977369	12·00 18·657720 ——— $\overline{7}$·342280			0·009492	0° 32′ 38″
150	2·176091	4·352182 6·022631 ——— $\overline{2}$·329551	13·056546 18·657720 ——— $\overline{6}$·398826			0·021358 0·000003 ——— 0·021361	1° 13′ 25″
192·80	2·285107	4·570214 6·022631 ——— $\overline{2}$·547583	13·710642 18·657720 ——— $\overline{5}$·052922			0·035284 0·000011 ——— 0·035295	2° 1′ 17″
200	2·301030	4·602060 6·022631 ——— $\overline{2}$·579429	13·806180 18·657720 ——— $\overline{5}$·148460			0·037969 0·000014 ——— 0·037983	2° 10′ 31″
250	2·397940	4·795880 6·022631 ——— $\overline{2}$·773249	14·387640 18·657720 ——— $\overline{5}$·729920			0·059327 0·000054 ——— 0·059381	3° 23′ 54″
300	2·477121	4·954242 6·022631 ——— $\overline{2}$·931611	14·862726 18·657720 ——— $\overline{4}$·205006	24·77121 31·505485 ——— $\overline{7}$·265725		0·085430 0·000160 ——— 0·085590	4° 53′ 31″
492·80	2·692671	5·385342 6·022631 ——— $\overline{1}$·362711	16·156026 18·657720 ——— $\overline{3}$·498306	26·92671 31·505485 ——— $\overline{5}$·421225	37·697394 44·116769 ——— $\overline{7}$·580625	0·230521 0·003150 0·000026 ——— 0·233697	13° 9′ 14″

Fig. 9.

23
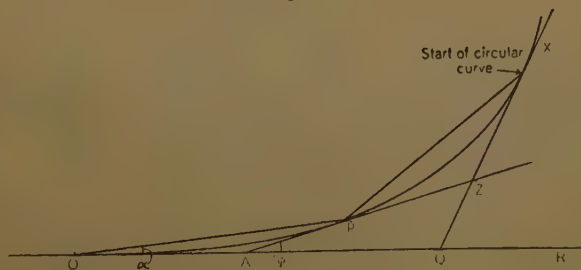
$$\phi = 0.256290$$
$$= 14° 41' 4''.$$

From the calculated Table, $\alpha = 4° 53' 31''$

Hence the back angle $= 9° 47' 33''$

The deflexion-angles from the tangent-line Ap must now be computed and tabulated as in Table B.

TABLE B.

l	Osculating-circle deflexion.	Spiral deflexion.	Total backward deflexion from Ap.	Total forward deflexion from Ap.
0	14° 41' 18''	4° 53' 31''	9° 47' 47''	—
50	12° 14' 25''	3° 23' 54''	8° 50' 31''	—
100	9° 47' 32''	2° 10' 31''	7° 37' 01''	—
150	7° 20' 39''	1° 13' 25''	6° 07' 14''	—
200	4° 53' 46''	0° 32' 38''	4° 21' 08''	—
250	2° 26' 53''	0° 08' 9''	2° 18' 44''	—
Change-point				
300	0° 00' 00''	0° 00' 00''	0° 00' 00''	0° 00' 00''
350	2° 26' 53''	0° 08' 9''	—	2° 35' 02''
400	4° 53' 46''	0° 32' 38''	—	5° 26' 24''
450	7° 20' 39''	1° 13' 25''	—	8° 34' 04''
492·80	9° 26' 23''	2° 1' 17''	—	11° 27' 40''

Sufficient information has now been obtained to enable the transition to be set out, and the deflexions for the circular portion must next be calculated.

Now $p\hat{Z}X = Z\hat{A}Q + (180° - Z\hat{Q}R)$ (see *Fig. 9*). But $Z\hat{Q}R =$ deviation taken up by first half of transition, and $Z\hat{A}Q =$ deviation reached after 300 feet.

Hence
$$p\hat{Z}X = 14° 41' 4''$$
$$+ 180°$$
$$= 194° 41' 4''$$
$$- 39° 37' 27''$$
$$= 155° 3' 37''$$

Again, $p\hat{X}Z = 180° - (X\hat{p}Z + p\hat{Z}X).$

From Table B,
$$X\hat{p}Z = 11° 27' 40''$$
$$155° 3' 37''$$
$$166° 31' 17''$$

Subtracting from 180°, $p\hat{X}Z = 13° 28' 43''.$

The theodolite may then be set up at X, sighted back on to p, and turned through $p\hat{X}Z$ to obtain the alignment of the tangent QX.

The deviation to be taken up by the circular arc
$$= 103° 39' 00''$$
$$- 79° 14' 54''$$
$$= 24° 24' 06''.$$

The radius of the circular arc will be the minimum radius attained by the transition, that is, 356·29 feet.

The angle subtended by a 50-foot chord at the centre of this circle

$$= 2 \sin^{-1} \frac{25}{356·29}$$
$$= 2 \times (4° \ 1' \ 25'')$$
$$= \qquad 8° \ 2' \ 50''$$

Hence there must be three 50-foot chords and a chord subtending an angle of :—

$$
\begin{array}{l}
24° \ 24' \ 06'' \\
- \ 24° \ \ 8' \ 30'' \ [3 \times (8° \ 2' \ 50'')] \\
\hline
\quad 0° \ 15' \ 36''
\end{array}
$$

The length of this chord will be

$$\frac{15' \ 36''}{8° \ 2' \ 50''} \times 50$$
$$= 1·615 \text{ feet.}$$

The following Table gives the necessary information for setting-out the circular curve.

Distance from point X : feet.	Polar deflexion-angle.
50	4° 1' 25''
100	8° 2' 50''
150	12° 4' 15''
151·615	12° 12' 03''

To complete the calculations the shift and the tangent-distance must be determined.

To find X :—

$$\phi = 39° \ 37' \ 27'' = 0·691573 \text{ radian.}$$
$$\log \phi = \overline{1}·839838$$
$$\log \sqrt{\phi} = \overline{1}·919919$$
$$\log m = 2·772755$$
$$\overline{}$$
$$\log m\sqrt{\phi} = 2·69267, \text{ whence } m\sqrt{\phi} = 492·804 \text{ feet.}$$

$$\log 5 \ \underline{|\,2} = \log 10 \ = 1·000000$$
$$\log 9 \ \underline{|\,4} = \log 216 \ = 2·334454$$
$$\log 13 \ \underline{|\,6} = \log 9360 = 3·971276$$

$2 \log \phi$	$4 \log \phi$	$6 \log \phi$	
$\overline{1}·679676$	$\overline{1}·359352$	$\overline{1}·039028$	
$2·692674$	$2·692674$	$2·692674$	(add log $m\sqrt{\phi}$)
$2·372350$	$2·052026$	$1·731702$	
$1·000000$	$2·334454$	$3·971276$	(subtract log of respective denominators)
$1·372350$	$\overline{1}·717572$	$\overline{3}·760426$	
$23·570$	$0·522$	$0·006$	(antilogs)

Hence
$$X = 492 \cdot 804 \text{ (1st term)}$$
$$- \quad 23 \cdot 57 \quad \text{(2nd term)}$$

$$469 \cdot 234 \text{ (1st term--2nd term)}$$
$$0 \cdot 522$$
$$- \quad 0 \cdot 006$$

$$= 469 \cdot 750 \text{ feet.}$$

To find Y :—

log 3	$= 0 \cdot 477121$
log 7 $\mid \underline{3}$ = log 42	$= 1 \cdot 623249$
log 11 $\mid \underline{5}$ = log 1320	$= 3 \cdot 120574$

$\log \phi$	$3 \log \phi$	$5 \log \phi$	
$\overline{1} \cdot 839838$	$\overline{1} \cdot 519514$	$\overline{1} \cdot 199190$	
$2 \cdot 692674$	$2 \cdot 692674$	$2 \cdot 692674$	(add log $m \sqrt{\overline{\phi}}$)
$2 \cdot 532512$	$2 \cdot 212188$	$1 \cdot 891864$	
$0 \cdot 477121$	$1 \cdot 623249$	$3 \cdot 120574$	(subtract log of respective denominators)
$2 \cdot 055391$	$0 \cdot 588939$	$\overline{2} \cdot 771290$	
$113 \cdot 603$	$3 \cdot 881$	$0 \cdot 059$	(antilogs)

Hence
$$Y = 113 \cdot 603 \text{ (1st term)}$$
$$- \quad 3 \cdot 881 \text{ (2nd term)}$$

$$109 \cdot 722 \text{ (1st term--2nd term)}$$
$$+ \quad 0 \cdot 059 \text{ (3rd term)}$$

$$= 109 \cdot 781$$

To find the shift :—

$$S = Y - R(1 - \cos \phi) \text{ (Formula (12), p. 339).}$$
$$\cos \phi = 0 \cdot 770244$$
$$1 - \cos \phi = 0 \cdot 229756$$
$$\log (1 - \cos \phi) = \overline{1} \cdot 361267$$
$$\log R = 2 \cdot 551806$$

$$\log R(1 - \cos \phi) = 1 \cdot 913073$$
$$R(1 - \cos \phi) = 81 \cdot 860$$
$$S = 109 \cdot 781$$
$$- \quad 81 \cdot 860$$

$$27 \cdot 921$$

To find the tangent-distance :—

$$(R + S) \tan \frac{\triangle}{2} + (X - R \sin \phi) \text{ (Formula (11), p. 339).}$$

In the present curve,

$$\frac{\triangle}{2} = 51° \ 49' \ 30''$$

$$\phi = 39° \ 37' \ 27'' \text{ for each transition.}$$

$$R = 356\cdot292$$
$$S = \ \ 27\cdot921$$

$$R + S = 384\cdot213$$

$$\log (R + S) = 2\cdot584572$$
$$\log \tan \frac{\triangle}{2} = 0\cdot104458$$

$$\log \left\{ (R + S) \tan \frac{\triangle}{2} \right\} = 2\cdot689030$$

whence

$$(R + S) \tan \frac{\triangle}{2} = 488\cdot686 \text{ feet.}$$

$$\log R = \ \ 2\cdot551806$$
$$\log \sin \phi = \ \ \overline{1}\cdot804650$$

$$\log R \sin \phi = \ \ 2\cdot356456$$

whence

$$R \sin \phi = 227\cdot225 \text{ feet}$$

$$X = 469\cdot75$$
$$R \sin \phi = 227\cdot23$$

$$X - R \sin \phi = 242\cdot52 \text{ feet}$$

Hence the total tangent-distance

$$= 488\cdot69$$
$$+ \ 242\cdot52$$

$$= 731\cdot21 \text{ feet.}$$

APPENDIX II.

It is required to set out a curve with lemniscate transitions in flat open country and to comply with the following standard. :—

(a) Speed-standard 50 miles per hour.
(b) Maximum permissible value of centrifugal ratio $B = 0.2$.
(c) Maximum permissible value of rate of change of centripetal acceleration $C = 1$ foot per second per second per second.
(d) Deviation between approach-straights $= 31°\ 00'\ 00''$.

By applying formula (15), $11.4CD^2 = V^3$, the length of unit chord may at once be determined.

$$11.4 \times 1 \times D^2 = 50^3,$$

whence $\hspace{4cm} D = 105.08$ feet.

The minimum theoretical radius is given by formula (6),

$$V^2 = 14.969BR,$$
$$50^2 = 14.969 \times 0.2R$$

whence $\hspace{4cm} R = 835.06$ feet.

If the curve has to be wholly transitional, then

$$\alpha = \frac{\phi}{6} = 5°\ 10'\ 00''.$$

Substituting in equation (16), $\rho = 10.3648D\sqrt{\sin 2\alpha}$

$$= 10.3648 \times 105.08\sqrt{\sin 10° - 20'},$$
$$= 461.28 \text{ feet}.$$

If the curve is wholly transitional, then, substituting in $\rho R = 35.8097D^2$,

$$461.28R = 35.8097 \times 105.08^2,$$

which gives $\hspace{4cm} R = 857.19$ feet.

Since this value is larger than the minimum allowable, the curve can be wholly transitional. It will be assumed, however, that as the country is open and flat the engineer decided to increase R to a large value, but to keep the scale of the transition and therefore the speed-standard unchanged. It will be taken that the minimum value decided on for R is 1,500 feet.

The value of ρ corresponding to this is given by

$$\rho R = 35.8097D^2$$

whence $\hspace{3cm} \rho = \dfrac{35.8097 \times 105.08^2}{1500}$

$$= 263.60 \text{ feet}.$$

The final value of α and hence ϕ for the transition part of the curve may now be found from formula (13),

$$\rho = 3R \sin 2\alpha,$$

whence $\hspace{3cm} \sin 2\alpha = \dfrac{263.60}{3 \times 1500}$

which gives $\hspace{3cm} 2\alpha = 3°\ 21'\ 30''$

and $\hspace{4cm} \alpha = 1°\ 40'\ 45''$

$$\phi = 5°\ 02'\ 15''$$

It is now necessary to decide on a suitable value of α for the first chord; this value of α should be such that the subchord length should assume a reasonable proportion. It is noticed that D, the length of unit chord which subtends a polar deflexion-angle of 16', is 105·08 feet. Thus if for setting-out purposes the first value of α is taken as 4' the first value of l will be very nearly $\sqrt{\dfrac{4}{16}} \times 105·08 = 52·54$ feet, a very convenient figure. As the deflexion of the transition is small it will be quite safe to increase α exactly as the square of the arc lengths desired. Hence convenient values for α will be 4', 4' $\times (2)^2$, 4' $\times (3)^2$, 4' $\times (4)^2$, 4' $\times (5)^2$ and so on.

It is now necessary to calculate the values of ρ corresponding to these decided values of α. This can be done by using formula (16), $\rho = 10·3648D\sqrt{\sin 2\alpha}$

$$\log 10·3648 = 1·015561$$
$$\log D = \log 105·08 = 2·021520$$
$$\log 10·3648D = 3·037081$$

This is the figure which must in Table C be added to the values of $\frac{1}{2}\log \sin 2\alpha$.

<div align="center">TABLE C.</div>

1	2	3	4	5
α	2α	$\log \sin 2\alpha$	$\frac{1}{2}\log \sin 2\alpha$	ρ
4'	8'	$\bar{3}·366816$	$\bar{2}·683408$ 3·037081 —————— 1·720489	52 ·54
4 $\times 2^2 = 16'$	32'	$\bar{3}·968870$	$\bar{2}·984435$ 3·037081 —————— 2·021516	105·08
4 $\times 3^2 = 36'$	1° 12'	$\bar{2}·321027$	$\bar{1}·160514$ 3·037081 —————— 2·197595	157·61
4 $\times 4^2 = 1° 04'$	2° 08'	$\bar{2}·570836$	$\bar{1}·285418$ 3·037081 —————— 2·322499	210·14
1° 40' 45"	3° 21' 30"	$\bar{2}·767733$	$\bar{1}·383867$ 3·037081 —————— 2·420948	263·60

The only column which is not self-explanatory in the above table is column 4. The first figure in each line of column 4 is obtained by halving column 3. To this result is added the logarithm of 10·3648D. The antilogarithm = ρ is entered up in column 5. Columns 1 and 5 of Table C are sufficient, if the polar-ray method of setting-out is adopted, but if this is either not possible or thought unsatisfactory, Table D permits either the offset method or the method of chaining continuously round the curve to be adopted.

TABLE D.

1	2	3	4	5	6	7	8	9	10	11
Station point.	α	log sin α	log cos α	x	y	x_1	y_1	Subchord²	Sub-chord $=l$	Σl
1	4′	$\overline{3}$·065786 1·720489 $\overline{2}$·786275	0·00	52·54	0·06	52·54	0·06		52·54	52·54
2	16″	$\overline{3}$·667845 2·021516 $\overline{1}$·689361	$\overline{1}$·999995 2·021516 2·021511	105·08	0·49	52·54	0·43	2760·45 0·18 2760·63	52·54	105·08
3	36′	$\overline{2}$·020021 2·197595 0·217616	$\overline{1}$·999976 2·196595 2·196571	157·61	1·65	52·53	1·16	2759·40 1·35 2760·75	52·54	157·62
4	1° 04′	$\overline{2}$·269881 2·322499 0·592380	$\overline{1}$·999925 2·322499 2·322424	210·10	3·91	52·49	2·26	2755·20 5·11 2760·31	52·54	210·16
5	1° 40′ 45″	$\overline{2}$·466905 2·420948 0·997853	$\overline{1}$·999813 2·420948 2·420761	263·49	7·72	53·39	4·35	2850·49 18·92 2869·41	53·57	263·73

If Table D is compiled there is no need to insert column 5 of Table C. Columns 3 and 4 of the Table D are obtained by adding log ρ (as found from column 4 of Table C) for the particular value of α to log sin α and log cos α respectively. Columns 5 and 6 are the antilogarithms of the results obtained in columns 4 and 3 respectively. Columns 7 and 8 are obtained by subtracting from the values in columns 5 and 6 for the particular line, the values in the line immediately above. In column 9 are added the squares of the values in columns 7 and 8, whilst the values in column 10 are the square roots of those in column 9. The values given in column 10 represent the sub-chords or lengths of chord between the polar rays determined by the values in column 2. Thus, in setting-out, the lengths will be run continuously from each previous station-point to intersect the polar ray for the next station-point. If it is desired to use the offset method of setting-out, columns 7, 8, 9, and 10 may be omitted.

As far as Table D goes, the square law can be seen to hold with good accuracy, and this law may without introducing appreciable error be assumed to hold up to a polar deflexion of about 3°, after which the tabular method of calculation must be applied. Actually for a polar deflexion of 3° 16′ and a speed-standard of 50 miles per hour the error is 0·88 inch in Σl, and is proportional to the chord-length for other speed-standards. This approximation is in fact Mr. Criswell's formula, except that he employed the American degree system, whereas in the present example Professor Royal-Dawson's unit chord system has been adopted.

For the whole curve $\triangle = 31° \, 00' \, 00''$

,,   two transitions $\phi = 10° \, 04' \, 30''$

,,   circular arc, deviation $= 20° \, 55' \, 30''$

Deviation for 50-foot chord on circular arc

$$= \sin^{-1}\frac{25}{1500}$$
$$= 0° \, 57' \, 18''$$

Hence the number of 50-foot chords required for the whole circular arc

$$= \frac{20° \, 55' \, 30''}{2 \times (57' \, 18'')}$$
$$= 10\cdot9555,$$

so that the circular arc will consist of 10 chords of 50 feet and one of $50 \times 0\cdot9555$ $= 47\cdot78$ feet.

Table E will allow the circular arc to be set out.

TABLE E.

Station point.	Sub-chord length.	Deviation from tangent at station-point 5.		
6	50	0°	57'	18''
7	50	1°	54'	36''
8	50	2°	51'	54''
9	50	3°	49'	12''
10	50	4°	46'	30''
11	50	5°	43'	48''
12	50	6°	41'	06''
13	50	7°	38'	24''
14	50	8°	35'	42''
15	50	9°	33'	00''
16	47·78	10°	27'	45''

To set out the circular arc the theodolite should be shifted to station-point 5 and sighted back on to 0. If it is then turned through $2\alpha = 3° \, 21' \, 30''$ and transited it will be tangent to the curve at station-point 5, and the above deflexions can then be set out from this tangent line.

To find the Shift.—

From formula (12),

$$S = Y - R(1 - \cos\phi)$$
$$\cos\phi = 0\cdot996137$$
$$1 - \cos\phi = 0\cdot003863$$
$$\log(1 - \cos\phi) = \bar{3}\cdot586925$$
$$\log R = 3\cdot176091$$

whence $\log\{R(1 - \cos\phi)\} = 0\cdot763016$

$$R(1 - \cos\phi) = 5\cdot79$$

Therefore $S = 7\cdot72$
$$- 5\cdot79$$
$$= 1\cdot93 \text{ feet.}$$

To find the Tangent-Distance.—

From formula (11), the tangent-distance $= (R + S) \tan \dfrac{\triangle}{2} + (X - R \sin \phi)$

In the present curve

$$\frac{\triangle}{2} = 15^\circ\ 30'\ 00''$$
$$\phi = 5^\circ\ 02'\ 15''\ \text{for each transition.}$$
$$R = 1,500$$
$$S =\quad\ 1\cdot93$$
$$R + S = 1501\cdot93$$

$$\log (R + S) =\quad 3\cdot176650$$
$$\log \tan \frac{\triangle}{2} =\quad \overline{1}\cdot442988$$
$$\log (R + S) \tan \frac{\triangle}{2} =\quad 2\cdot619638$$

$$\therefore (R + S) \tan \frac{\triangle}{2} = 416\cdot522$$

$$\log R =\quad 3\cdot176091$$
$$\log \sin \phi =\quad \overline{2}\cdot943532$$
$$\log R \sin \phi =\quad 2\cdot119623$$

$$\therefore R \sin \phi = 131\cdot711$$
$$X = 263\cdot49$$
$$R \sin \phi = 131\cdot71$$

$$X - R \sin \phi = 131\cdot78\ \text{feet.}$$

Hence the total tangent-distance

$$=\quad 416\cdot52$$
$$+\ 131\cdot78$$
$$=\quad 548\cdot30\ \text{feet.}$$

Actually, in the above example, since α is less than 3° two assumptions could have been made. S (see *Fig. 5*) could have been determined directly from $S = \dfrac{Y}{4}$ and $R \sin \phi$ could have been assumed equal to $\dfrac{X}{2}$ which is the same as assuming that the transition portion of the curve is divided equally by the original tangent point of a circular arc having radius $= (R + S)$. Also it could have been assumed that α is directly proportional to the square of the length of transition.

Paper No. 5187.

"Wind-Pressure Experiments at the Severn Bridge."

By ALFRED BAILEY, M.Sc., Assoc. M. Inst. C.E. and NOEL DAVID
GEORGE VINCENT, B.Sc.

(Ordered by the Council to be published with written discussion.)[1]

TABLE OF CONTENTS.

INTRODUCTION.

THE investigation described in the present report forms a continuation of the work carried out by the late Sir Thomas Stanton, F.R.S., M. Inst. C.E., and described by him in a Paper[2] presented to The Institution of Civil Engineers in 1924.

The object of the experiments described in this Paper is to attempt to determine whether, in the case of a wind-gust striking a structure of large dimensions, the maximum average pressure on the structure is less than the maximum pressure indicated at a single point. Sir Thomas Stanton's experiments were made, first by means of a number of pressure-tubes placed on towers, 60 feet high, in Bushy Park, Teddington, and covering a total span of 350 feet, and afterwards on Tower bridge, London, which has a span of 225 feet. It was considered that the former site did not give representative results, and as regards the latter, the general conclusion arrived at was as follows : " . . . that in winds of moderate intensity, up to 50 miles per hour, the pressure on a large area during the passage of a gust is, in the majority of cases, appreciably less than that on a small area. There is, however, very definite evidence of the existence of gusts

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Fig. 1.

ELEVATION OF SEVERN BRIDGE.

up to 50 miles per hour in which the
variation in velocity over the front is so
small that, in the case of a structure in an
exposed position and having a span of less
than 250 feet, the pressure over the
whole area is sensibly equal to that at a
point in it. Obviously, if this condition
were found to obtain for gusts of still
higher intensity and for structures of
much greater span, no reduction in the
wind-pressure factor on account of lateral
variation of wind-speed could be justified.

It was considered desirable that these
experiments should be repeated on a
much larger span, and at the request of
the Steel Structures Committee of the
Building Research Board this work was
put in hand.

It was decided to carry out the ex-
periments on the Severn bridge, which
is over 4,000 feet long and carries a single-
track railway across the river Severn at
a point about ¾ mile above Sharpness,
Gloucestershire. This bridge, shown in
Fig. 1, is composed of twenty-one bow-
string girders, having spans varying from
134 feet to 324 feet, carried on twin
cylindrical piers. The bridge offers very
little obstruction to the wind and, as the
prevailing winds in the British Isles are
south-westerly and meet the bridge
almost exactly broadside, it was con-
sidered to meet all the requirements.
On the windward side of the bridge, the
land is reasonably flat and offers no major
obstruction to the oncoming wind. A
map of the site is given in *Fig. 2*.

SCHEME OF THE EXPERIMENTS.

The general scheme proposed was:
(a) to erect a number of fixed "Dines"
heads, with the pressure tubes facing the
prevailing wind, over as wide a wind
front as practicable, these being so inter-

Fig. 2.

MAP OF SITE.

connected as to give the average of their pressure-readings, and (b) to place an additional head at the centre of the group to give the pressure at this point. Since the two long spans near one end of the bridge have girders which are 40 feet high, whilst the remainder are about 17 feet high, it was decided to confine the tests to the shorter spans from which a total test length of 2,680 feet could be obtained. To cover this distance, ten " Dines " heads were erected at a height of 32 feet above the bridge platforms on tubular steel poles at approximately equal distances apart

Figs. 3.
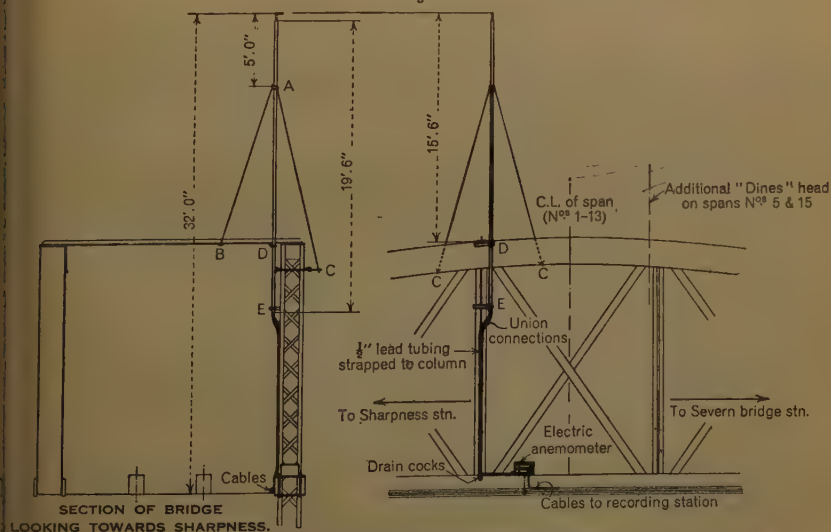
BRIDGE FIXINGS FOR " DINES " HEADS.

(*Fig. 1* and *Figs. 3*), and an additional head was erected near the centre of each group of five. Arrangements were then made to obtain simultaneous continuous records of the average pressure on each group of five heads and on each of the two additional heads.

APPARATUS : THE ELECTRIC ANEMOMETER.

As the usual method of transmitting the pressure to the recorder by piping could not be employed on account of the long distances, an electrical method of transmission was developed. The electric anemometer, which was finally found to be successful, is shown in *Figs. 4.* It

Figs. 4.
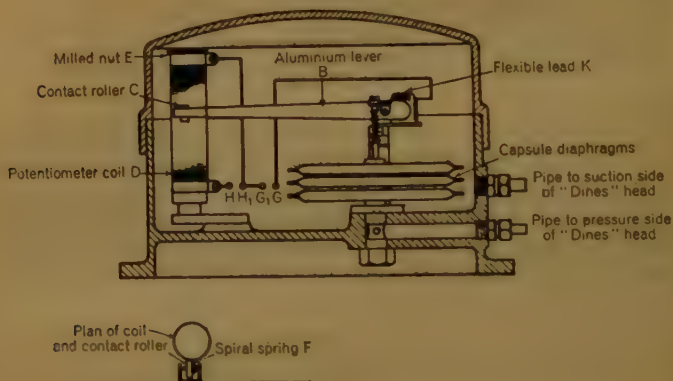
GENERAL ARRANGEMENT OF ELECTRIC ANEMOMETER.

consists of a pile of thin metallic capsules, 5 inches in diameter, made of corrugated sheets of German silver, 0·002 inch in thickness and soldered at the edges. The expansion of the capsules operates an aluminium lever, pivoted in jewel-bearings, by means of a double-pointed steel strut, one end of which rests in a cup on top of the capsules and the other end in a cup fixed to the lever, this lever giving a magnification of movement of about 20 to 1. At the free end of the lever is a small bracket carrying a pivoted stainless-steel roller, $\frac{1}{2}$ inch in diameter and $\frac{1}{8}$ inch wide on the face, which is pressed lightly on to a vertical potentiometer-coil. The coil is made by close winding No. 32 silk-covered "Eureka" wire on a brass former, previously treated with bakelite varnish. The ends of the coil are soldered to clips of "Eureka" strip which carry the connecting terminals, the outside of the coil then being treated with bakelite varnish which is scraped off over the narrow strip where contact with the roller takes place.

Positive electrical contact between the roller and the lever-arm is obtained by means of a hair-spring, and between the lever-arm and a fixed terminal by means of fine copper "pigtail."

The instrument is enclosed in a circular cast-iron box, with a close-fitting machined lid made airtight by grease on the joint.   Four terminals pass through the wall, two for supplying current to the coil and two for the potential leads to the recorder ; there are also two pipe connexions, one leading to the inside of the capsules and the other into the box, the former being connected to the pressure tube of the " Dines " head and the latter to the suction-tube, so that the pressure-difference is acting on the capsules.   Experiments with this instrument show that when a steady current of electricity is passed through the coil, the potential-difference between one end of the coil and the point of contact of the roller is proportional to the pressure-difference applied to the capsules, and continuous running over long periods shows that the calibration is well maintained.

In order to reduce to a negligible amount the possible effect of contact-

Fig. 5.
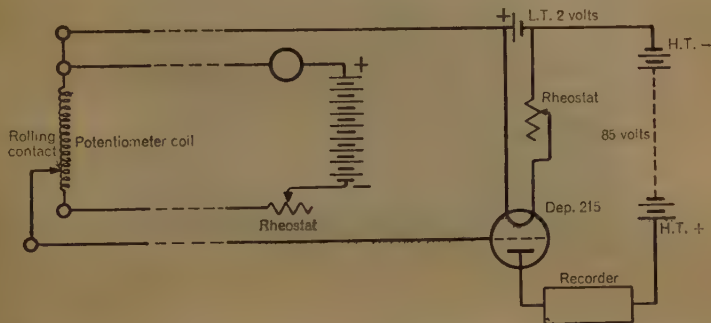
DIAGRAM OF CONNEXIONS FOR ELECTRIC ANEMOMETERS (RECORDER OPERATED BY A SINGLE INSTRUMENT).

resistance between the roller and the potentiometer-coil, and also the effect of variation in resistance of the potential-leads, the latter are connected at the recorder end to the grid-circuit of a small two-volt power-valve ; by this means the variations of potential can be transmitted over long distances without loss and can be determined by calibrating against the anode-current of the valve.

The final record is obtained by means of a milliammeter recorder, which, on calibration, gives the variation in anode-current as a function of the pressure-difference applied to the electric anemometer.   Also, since the potential-difference is proportional to the pressure-difference, the average of the pressure-differences for a series of anemometers can be recorded by connecting the potential points of a number of anemometers in series.

The scheme of wiring used to obtain the pressure-difference at a single instrument and the average pressure-difference of a group of instruments is shown respectively in *Figs. 5* and *6*.

Fig. 6.
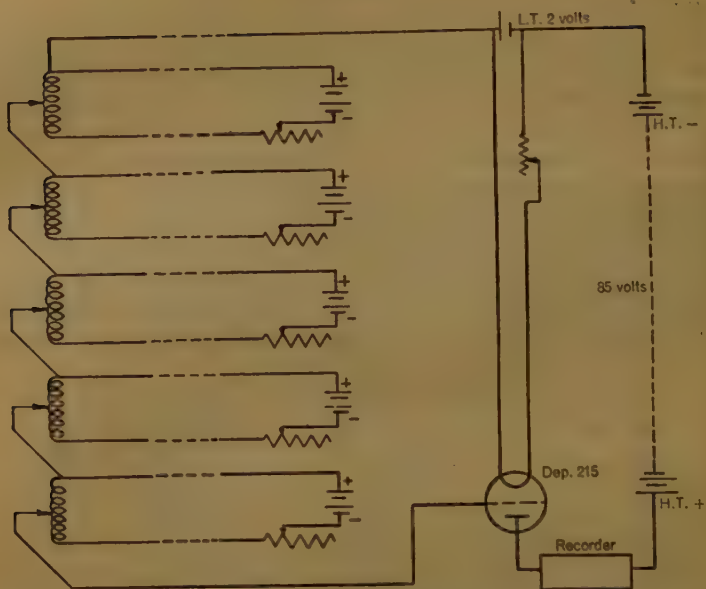
DIAGRAM OF CONNEXIONS FOR ELECTRIC ANEMOMETER (SEVERAL INSTRUMENTS
OPERATING IN SERIES).

THE LAYOUT AT THE SEVERN BRIDGE.

Fig. 7 shows one of the fixed " Dines " heads erected on the tubular-
steel pole. The opening of the pressure-tube was set to face a wind at
right angles to the bridge from the south-westerly direction, the clearance
above the top of the girder being about 15 feet. The bridge has a slight
fall towards the Sharpness end (about 1 in 150), but it was not considered
necessary to vary the lengths of the poles as the differences between the
end instruments and the mean level was only $\pm$ 10 feet. The pressure-
tube and the suction-holes of each of the " Dines " heads were connected
by copper tubing to an electric anemometer carried on the bottom boom
of the girder, as shown in *Fig. 8*, on which can also be seen the drain taps
used for keeping the connecting pipes free from water. At the middle
point of the length under observation an additional pole was erected
carrying a standard rotating " Dines " head (*Fig. 9*, facing p. 369)
connected at the base to a standard wind speed-and-direction recorder
intended to provide a continuous record of wind speed-and-direction for
correlation with the anemometer records.

Fig. 7.

THE FIXED "DINES" HEAD.

Fig. 8.
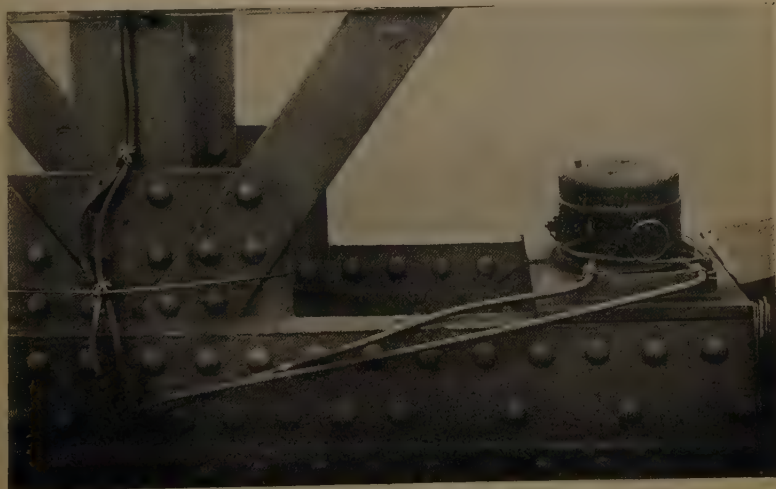
ELECTRIC ANEMOMETER CONNEXIONS.

Fig. 9.

THE STANDARD "DINES" HEAD.

Fig. 11.
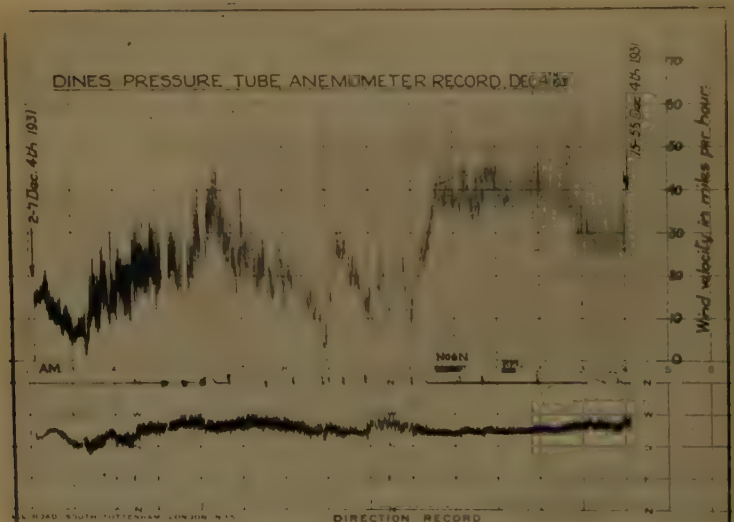
"DINES" HEAD RECORD.

The anemometer-records were obtained on a four-pen milliammeter recorder having a maximum range of 8 milliamps, and a chart-speed of 3 inches per minute. The instrument was housed, together with the necessary switchboards and electric accumulators, in a hut on the embankment near one end of the bridge.

Each electric-anemometer-coil was connected to its appropriate point on the switchboard in the recorder-hut by a twin 1/0·044 lead-covered rubber-insulated copper cable through which the necessary current was supplied from a battery of accumulators. Similar single wires were used to transfer the potential changes in the anemometers to the grid-circuits of the valves, and an additional single wire was laid on to a plug-point in each instrument-box for connexion to a portable telephone for use when calibrating the anemometers. All wires were carried along the platform of the bridge in wooden troughs.

The five anemometers covering one-half of the total length under test were connected in series by the potential leads, so that the recorder pen operating through this circuit gave the average pressure-difference of the five instruments. The five instruments covering the other half were similarly connected, the two remaining recorder pens being operated by the two single instruments at the middle point of each half.

CALIBRATION OF THE INSTRUMENTS.

The Anemometer.

The anemometers were calibrated in position by applying a known pressure-difference, measured on a water manometer, through the two drain taps (shown in *Fig. 8*, facing p. 368), the two upper taps being used to shut off connexion to the " Dines " head during this operation. A high-tension voltage of 85 was used in the anode-circuit of the valves and the filament-current was adjusted until an emission of 8 milliamps was obtained with zero grid-volts. The direction of the current in the potentiometer-coil of the anemometer was arranged so that when the arm was in its zero position, with no pressure-difference on the diaphragm, the maximum negative grid-voltage was applied to the valve and the anode-current was reduced to a low value. Any application of pressure-difference to the anemometer capsules raised the lever arm, thus reducing the negative grid potential and increasing the anode-current. Increments of pressure were applied in this way, and at each increment the recorder was operated. This gave the relation between anode-current and pressure-difference and from a previous determination of the valve-characteristic-curve (anode-current and grid-potential) the straight line connecting pressure-difference and grid potential was drawn, such as that shown on *Fig. 10* (p. 370). It was necessary to obtain this graph, since, in the case of the instruments which were connected in series, it was essential that they should all have the same ratio of grid-potential change to pressure-difference change. This

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was achieved, after the instruments had been calibrated, by suitably adjusting the currents in the potential coils.

Fig. 10.
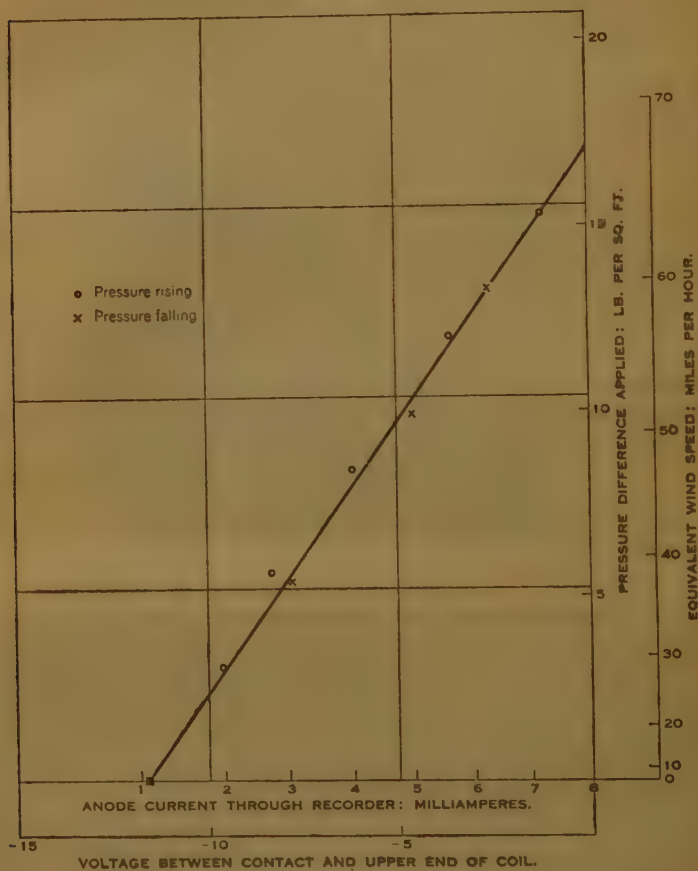
TYPICAL CALIBRATION CURVE.

The Fixed " Dines " Head.

Before dispatch to the bridge, the fixed " Dines " heads were calibrated in the wind-tunnel at various speeds up to 40 miles per hour. The well-known relation between pressure-difference and wind-velocity for the instrument is

$$p = k \times \tfrac{1}{2} \frac{p}{g} v^2,$$

where p denotes the pressure-difference in lb. per square foot,

ρ denotes the air density in lb. per cubic foot,

v denotes the wind velocity in feet per second,

$g = 32 \cdot 2$ feet per second per second,

and k is the instrument coefficient,

The mean value of k was found to be $1 \cdot 45$.

The effect of wind-direction was also investigated and it was found that the change in pressure-difference was negligible up to ± 20 degrees.

The Standard " Dines " Anemometer.

The standard " Dines " head was calibrated in the wind-tunnel and was found to have a coefficient (k) of $1 \cdot 48$.

Owing to the exposed position in which this instrument was placed on the Severn bridge, it was necessary to use a solution of glycerine in water in the float-chamber instead of water, to prevent freezing in winter. The rise of the float was calibrated against a standard water-gauge and was found to require a correction on the wind-velocity scale varying from zero to $3\frac{1}{2}$ per cent.

METHOD OF TEST AND RECORDS OBTAINED.

As the site of the tests was about 5 hours' journey from the National Physical Laboratory and it was impracticable to have observers permanently in attendance, arrangements were made with the Director of the Meteorological office to provide the Laboratory with advance information of the approach of south-westerly gales to this area.

For the purpose of the experiment only high-speed winds were of value, and it was not considered worth while to visit the bridge unless gust-speeds of about 40 miles per hour were expected. Further, to obtain satisfactory records it was necessary that the wind-direction should be within ± 20 degrees of normal to the bridge. These conditions considerably restricted the opportunities for obtaining records, and the first satisfactory wind occurred on the 4th December, 1931, when there was a south-westerly gale, with occasional gusts of nearly 60 miles per hour, the record obtained on this occasion from the Standard " Dines " Anemometer at the middle of the bridge being reproduced in *Fig. 11*. The gale lasted for about 3 hours, from noon to 3 p.m., during which period several series of anemometer records were taken. A typical portion of one of these records taken between 2.31 p.m. and 2.36 p.m., when the gale was at its highest, has been analysed and replotted in the form of wind-velocity curves in *Fig. 12* (p. 372), the four curves having been plotted to the same base line for easy comparison and an additional curve showing the mean value of the two groups having also been added. It should be noted that since
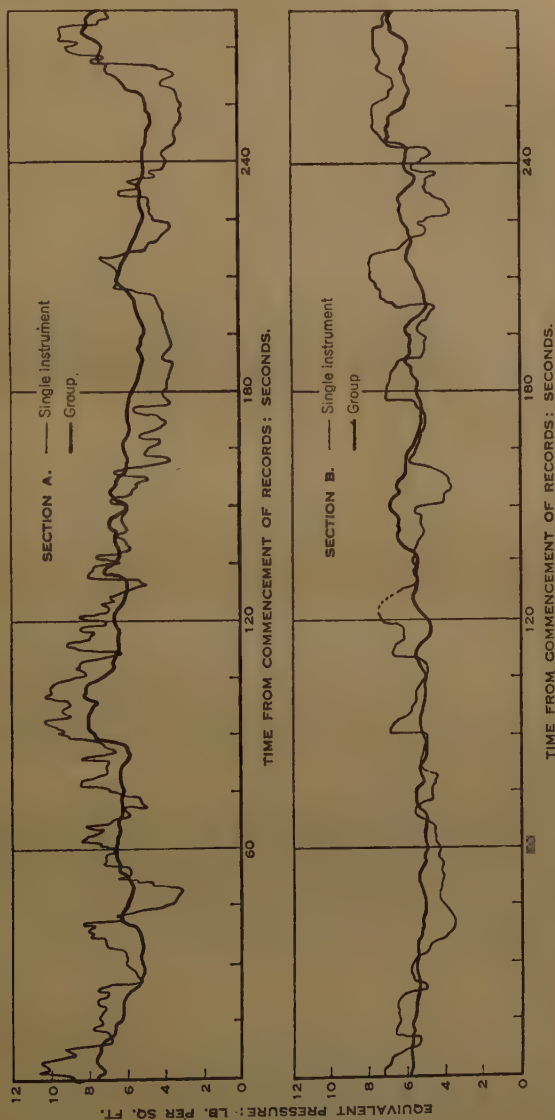
24*

Fig. 12.
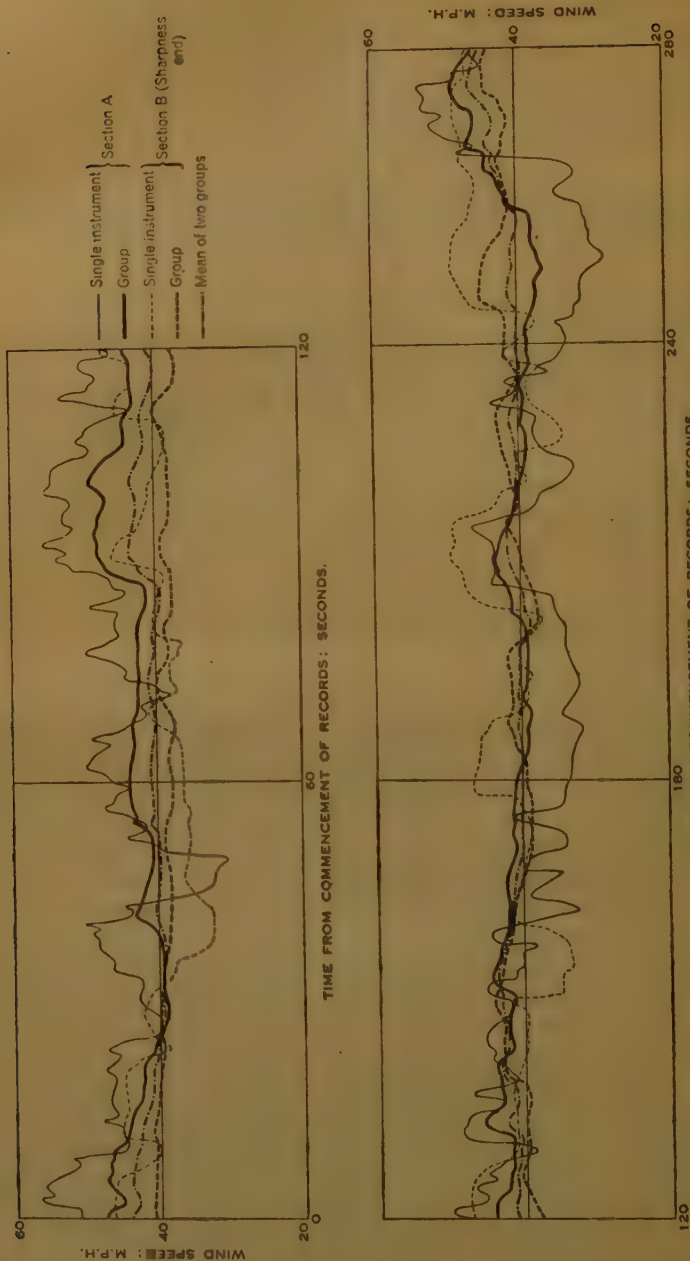
TYPICAL RECORDS OF WIND-VARIATIONS AT THE SEVERN BRIDGE.
(4TH DECEMBER, 1931.)

the group-record gives the average of the pressures at the five instruments, the velocity-curve is the root-mean-square of the velocities and is not necessarily the true average-velocity.

Fig. 13.

REDUCTION OF 4TH DECEMBER, 1931 RECORDS TO EQUIVALENT PRESSURE-SCALE.
(Period 2.31 p.m. to 2.36 p.m., 4th December, 1931.)

¹ A. Bailey, "Wind Pressures on Buildings." Inst. C.E. Selected Engineering Paper No. 139, 1933, p. 5.

The intensity of pressure of the wind on a fully exposed surface has been shown by Sir Thomas Stanton [1] to be given by :

$$P = 0 \cdot 0034 V^2,$$

where P denotes the intensity of pressure in lb. per square foot,
and V denotes the wind velocity in miles per hour.

Applying the formula to the values given in *Fig. 12* two sets of curves have been drawn showing the relation between the average intensity of pressure over each half of the bridge and the intensity of pressure at the middle of each half (*Fig. 13*).

For the Tower bridge tests, the graphs of average pressure and of pressure at a point were very similar in that the main peaks and depressions of the wind were reproduced in each, indicating that a gust striking the bridge affected all points simultaneously. The graphs obtained in the present tests show that this does not apply to spans of the order of 1,340 feet, as although the long period changes of wind-speed indicated by the single instrument are generally followed to some extent by the average, the short period peaks and depressions seldom show an effect over the whole width. Similarly, on comparing the graphs of the two single instruments, which are 1,340 feet apart, the velocity-curves (*Fig. 13*) seldom show any similarity, and in fact, one is frequently rising when the other is falling. It is concluded, therefore, that the width of action of a gust is normally less than 1,340 feet under the conditions of the present tests.

Under these circumstances any relation between the pressure at a point and the average pressure over the whole span must be fortuitous, but in order to obtain some numerical data on this point a comparison has been made of the peak-pressures which appear on both the single instrument and the corresponding group within a period of 5 seconds of each other. The ratio of the two values has been calculated and the results are shown plotted against peak wind-speed indicated at the single instrument on *Figs. 14* and *15*.

From these graphs it would appear that the ratio has a general tendency to fall as the wind-speed increases. At speeds above 50 miles per hour indicated at the centre, there was only one occasion in each section when the average pressure was over 90 per cent. of that at the centre, and if the rate of fall is maintained the ratio would be substantially less at 70 or 80 miles per hour.

MODIFIED SCHEME OF TESTS.

Following these experiments it was considered advisable to make an attempt to determine the maximum width of action of a gust, and additions were made to the connexions in the recorder-hut so that any one instrument in each of the groups of five could be used as a single instrument. The

Fig. 14.
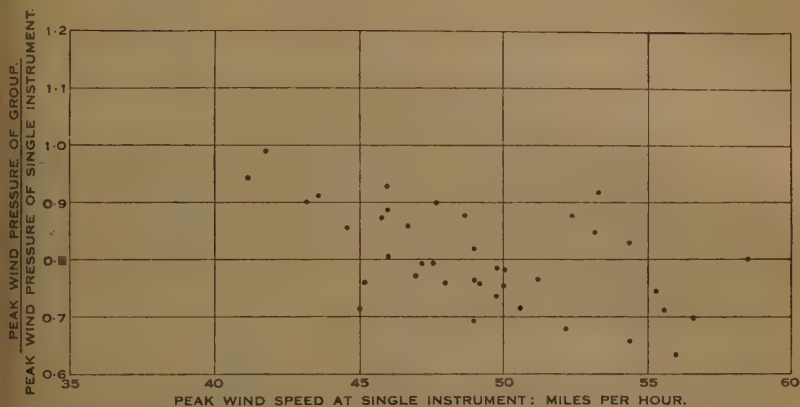
CORRELATION OF AVERAGE PRESSURE WITH PRESSURE AT A SINGLE POINT.
(4TH DECEMBER, 1931 : SECTION A, LYDNEY END.)

Fig. 15.
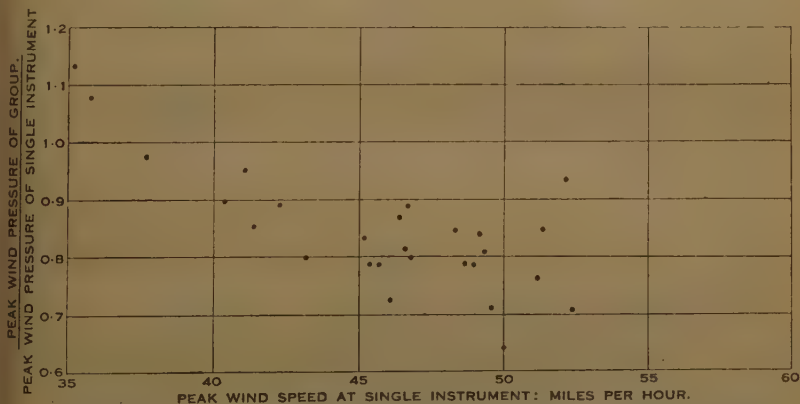
CORRELATION OF AVERAGE PRESSURE WITH PRESSURE AT A SINGLE POINT.
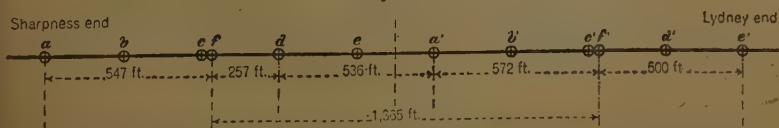(4TH DECEMBER, 1931 : SECTION B, SHARPNESS END.)

Fig. 16.

DIAGRAM SHOWING RELATIVE POSITIONS OF INSTRUMENTS USED IN TEST.

scheme was to obtain simultaneous records of four single instruments, the positions of two of these being varied occasionally. In this way it would be possible to have two instruments covering spans varying from 268 feet up to the maximum in intervals of 268 feet, and if a gust of wind affected the entire span under test, the two instruments would give records which would be in phase, although not necessarily of the same magnitude.

This change was made early in 1932, and *Fig. 16* shows the relative positions and identification marks of the instruments used in the test. On the early morning of the 14th January, 1934, suitable gale conditions were obtained, the wind veering from S.S.E. at midnight, to S.W. at about 5.0 a.m., accompanied by an increase of speed, and under these conditions it was possible to obtain pressure-records.

Portions of these records have been plotted to a scale corresponding to the intensity of pressure which would be produced on a square plate placed in the position of the fixed " Dines " head, as described above. The results are shown on *Figs. 17, 18* and *19* (pp. 377 *et seq.*)

An examination of these records shows that, even at the shortest distance available (257 feet on section B, *Fig. 18*), there is no regular agreement between the pressures at the two ends of the span ; there are occasional gust-peaks, such as those indicated by " X " in *Figs. 17, 18* and *19*, which are similar and appear either simultaneously or within a few seconds of each other, but this similarity in wind-structure is not maintained and may be due to fortuitous circumstances rather than to uniformity of conditions.

It will be noted that, in general, the pressures at the Sharpness end of the bridge are lower than those at the Lydney end, this being no doubt due to the screening effect of Sharpness point (*Fig. 2*, p. 365).
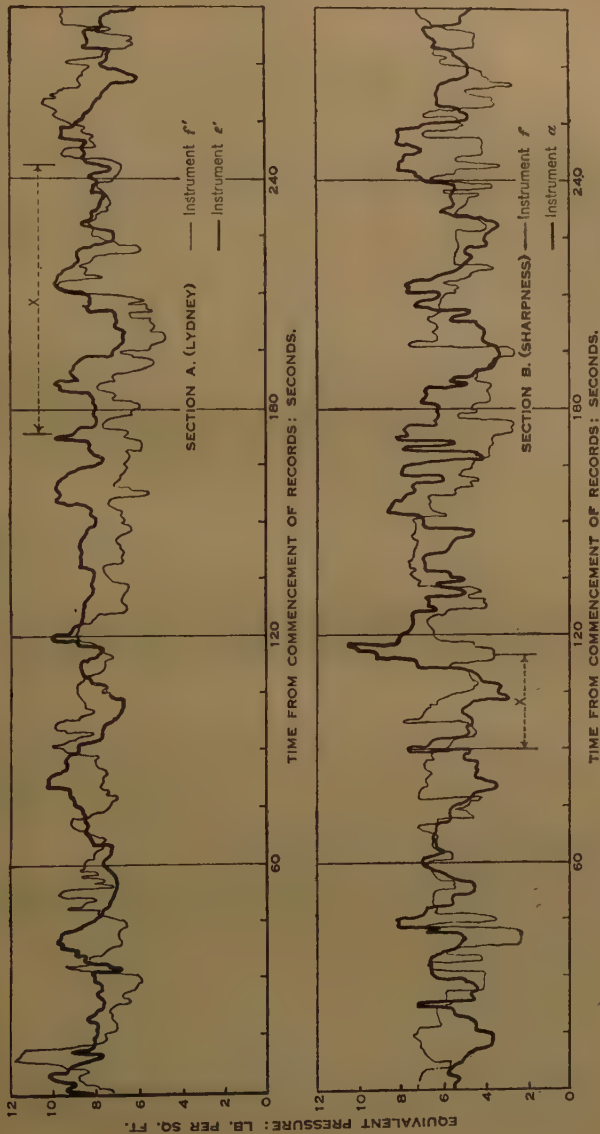
Conclusions.

The conclusion to be drawn from these results appears to be that, so far as this site is concerned, the wind-front is generally composed of a series of eddies of smaller magnitude than 250 feet which are more or less independent of each other. The likelihood of the average pressure over a wide front being equal to the maximum pressure at any one point depends, therefore, upon the probability of a number of eddies striking simultaneously, which is in agreement with the results obtained when the instruments were connected in groups.

Experiments have been carried out in America on somewhat similar lines [1] and in these, eight instruments were spread over a wind front of 480 feet and at a height of 50 feet, and a similar series of five instruments was placed on a vertical line covering 250 feet. An independent con-

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[1] R. H. Sherlock and M. B. Stout, " Picturing the Structure of the Wind." *Civil Engineering*, vol. 2 (1932), p. 358.
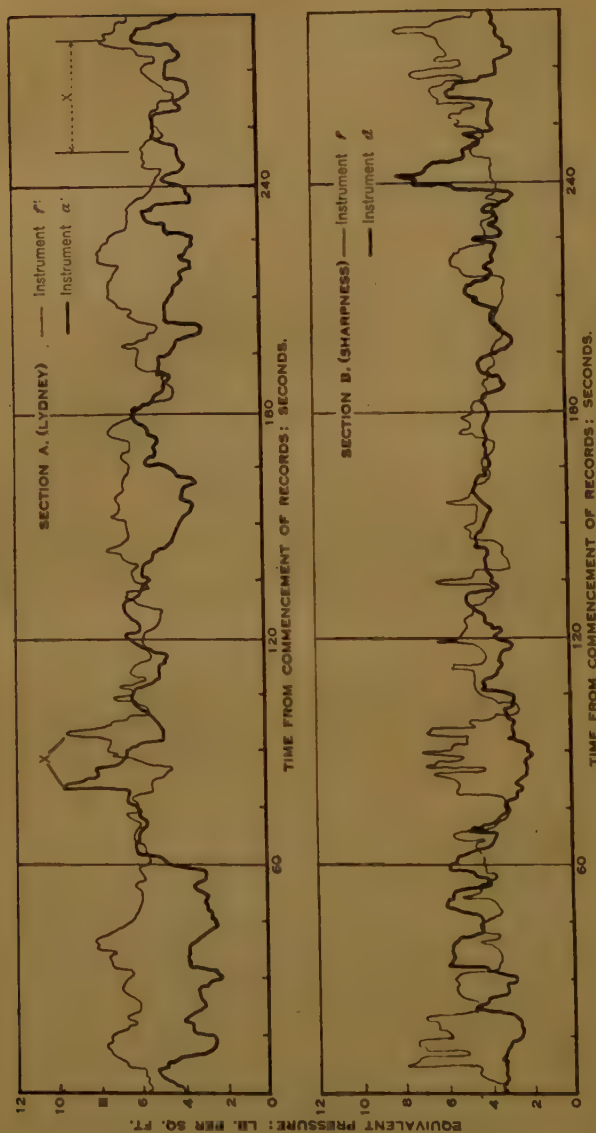
Fig. 17.

SECTION A. (LYDNEY)

—— Instrument *f'*
—— Instrument *e'*

TIME FROM COMMENCEMENT OF RECORDS: SECONDS.

SECTION B. (SHARPNESS) —— Instrument *f*
—— Instrument *a*

TIME FROM COMMENCEMENT OF RECORDS: SECONDS.

RECORDS REDUCED TO EQUIVALENT INTENSITY OF PRESSURE ON A SQUARE
PLATE.

(Period 5.08 a.m. to 5.12¾ a.m., 14th January, 1934.)

EQUIVALENT PRESSURE: LB. PER SQ. FT.
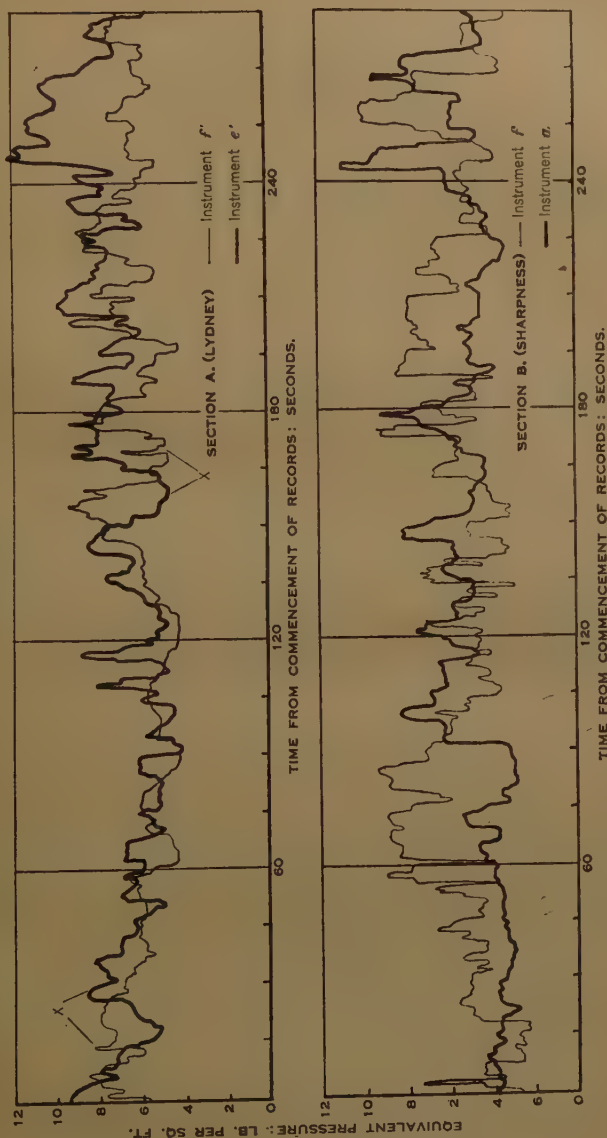
Fig. 18.

RECORDS REDUCED TO EQUIVALENT INTENSITY OF PRESSURE ON A SQUARE
PLATE.

(Period 6.02½ a.m. to 6.07 a.m., 14th January, 1934.)

Fig. 19.

SECTION A. (LYDNEY)
—— Instrument *f'*
—— Instrument *e'*

TIME FROM COMMENCEMENT OF RECORDS: SECONDS.

SECTION B. (SHARPNESS) —— Instrument *f*
—— Instrument *a.*

TIME FROM COMMENCEMENT OF RECORDS: SECONDS.

EQUIVALENT PRESSURE:: LB. PER SQ. FT.

RECORDS REDUCED TO EQUIVALENT INTENSITY OF PRESSURE ON A SQUARE
PLATE.
(Period 6.13 a.m. to 6.17¾ a.m., 14th January, 1934.)

tinuous reading of wind velocity was obtained from each instrument, and from these readings iso-velocity lines were drawn from which a " picture " of the wind structure was obtained. A conclusion drawn from these tests is " that the combination of 50- and 60-foot spacing with time-intervals of half-seconds is about the least refined combination which could have been used without losing all indication of the nature of the turbulent areas disclosed."

It appears, therefore, that the general conclusion drawn by Sir Thomas Stanton from the Tower bridge experiments (quoted at the beginning of this Paper), is also applicable to structures of much larger dimensions.

The Paper is accompanied by eleven sheets of diagrams, from which the Figures in the text have been prepared, and by fifteen photographs, from some of which the half-tone page-plate has been prepared.

Paper No. 5191.

"The Water-Supply of Kumasi in the Gold Coast Colony."

By CHRISTOPHER WILSON BROWN, O.B.E., M.C., and CECIL LEE HOWARD HUMPHREYS, T.D., MM. Inst. C.E.

(Ordered by the Council to be published in abstract form.)[1]

THE Paper deals with the preliminary work, the design, and the construction of a waterworks scheme for a future population of 50,000 in West Africa, and illustrates the difficulties which are peculiar to a tropical country, about which very little information is available.

An account is given of the attempts to forecast the probable growth of the population, the water-consumption for native quarters from a consideration of available records of other similar tropical towns, the run-off in jungle territory from gauge-readings taken on the Owabi river together with comparative records from the rivers Adra and Ankoni, and the evaporation in this tropical climate by means of small-scale experiments.

A full analysis is given of the capital cost (£216,000) of the works divided into their several portions. The relative costs of oil, electricity, and producer-gas as a source of power for the pumping plant are considered in detail, and it is shown how the decision to use producer-gas from wood fuel was reached despite the added difficulty of having to purify the scrubber-water before allowing it to flow back into the river below the dam.

Difficulties encountered when the treatment plant was first put into operation, partly because of decaying organic matter and partly because of manganese, are dealt with, as is the subsequent successful treatment of the water with lime, chloride of lime, and aluminium sulphate in the various stages of sedimentation and filtration, including automatic chlorination in the suction pipe to the pumps.

The works described include an earthen dam with a concrete core, forming an impounding reservoir on the Owabi river with a capacity above the lowest draw-off of 120,000,000 cubic feet; filtration and purification plant to deal with 1,000,000 gallons in each 24 hours; a pumping station which contains two four-stage horizontal-spindle turbine-pumps, driven by two 120/160-h.p. horizontal producer-gas engines, delivering 680 gallons per minute against a total head of 341 feet, and a 100/250-volt 40/26·5-ampere compound-wound generator for the lighting of the pump-house, offices, and bungalows; 6⅜ miles of 14-inch bitumen-lined rising main; a 18,000,000-gallon storage-reservoir built of mass concrete and with a

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[1] Copies of the Paper may be obtained on loan from the Loan Library of the Institution; a limited number of copies is also available, for retention by members, on application to the Secretary.

reinforced-concrete roof; and the distribution system in Kumasi. An account is given of the progress of the work, which was necessarily slow with the unskilled native labour available and of the difficulties that arose. The works were commenced in March, 1930, and were completed in March, 1934.

The Paper is accompanied by a photograph of the dam, seven drawings of the more important parts of the work, and a graph showing the effect of relative humidity, temperature, and wind-velocity on the evaporation.

ENGINEERING RESEARCH.

THE INSTITUTION RESEARCH COMMITTEE.

Committee on Bituminous Jointing Materials for Concrete.

IN August, 1938, the recommendations of the Sub-Committee on Reinforced Concrete Structures for the Storage of Liquids were published in the form of a Code of Practice,[1] in which mention was made of the use of bituminous compounds to render water-tight the joints in articulated structures. Although the Sub-Committee had instituted a research into the properties of bituminous materials for use in such joints, it was not considered that sufficient information was available at the time of publication of the Code to permit the formulation of specifications or recommendations thereon. The Sub-Committee accordingly recommended that a new Committee should be set up " to study bituminous and allied jointing materials for concrete and to make recommendations for their use in water-retaining or other structures." This Committee has now been formed with the following personnel :—

> W. H. GLANVILLE, D.Sc., Ph.D. (*Chairman*).
> Professor R. G. H. CLEMENTS, M.C.
> H. J. DEANE, B.E.
> OSCAR FABER, O.B.E., D.Sc.
> H. B. MILNER, M.A.
> H. C. RITCHIE.
> W. L. SCOTT.
> F. G. TURNER, B.Sc.

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[1] "Code of Practice for the Design and Construction of Reinforced-Concrete Structures for the Storage of Liquids." Obtainable from William Clowes and Sons, Ltd., 94, Jermyn Street, S.W.1. Price to members 1s. 6d. post free; to non-members 2s. 6d. post free.

The Committee will examine the reports of experimental work and the tentative specifications that have so far been produced, and will be able to continue the study of bituminous jointing materials by whatever methods may be found suitable. The terms of reference to the Committee are not restrictive, and it is intended to study materials for various types of joints in liquid-retaining and other concrete structures, including roads.

DEPARTMENT OF SCIENTIFIC AND INDUSTRIAL RESEARCH: REPORT FOR THE YEAR 1937–38[1].

The Report describes the progress of the many activities of the Department during the year; the net expenditure for the year was approximately £637,000, an increase of £55,000 over that of the previous year. In addition, the aggregate sum subscribed by industry through the research associations was nearly £235,000.

Many of the researches carried on by the Department have already been reviewed in the Journal during the last year in connexion with the Annual Reports on the work of the Building Research Board[2], the National Physical Laboratory[3], the Water Pollution Research Board[4], the Chemistry Research Board[5], the Forest Products Research Board[6], the Road Research Board[7], the William Froude Laboratory[8], and the Fuel Research Board[9]. The following notes are selected from parts of the Report dealing with other subjects of interest to engineers.

Arrangements have been made for the National Physical Laboratory to supervise and certify short-circuit tests on electrical switchgear up to capacities of 250,000 kVA. in the testing stations of the Association of Short Circuit Testing Authorities.

The Geological Survey has carried out widespread field work and the preparation of geological maps, the districts covered being listed in the Report, and studies of underground water-supplies have been extended; a memoir on the underground water-supply of the County of London has been published, and has been reviewed in the Institution Journal[10].

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[1] Published by H.M. Stationery Office, price 3s. 0d. net.
[2] Journal Inst. C.E., vol. 9 (1937–38), p. 345. (June 1938.)
[3] Ibid., vol. 9 (1937–38), p. 347. (June 1938.)
[4] Ibid., vol. 9 (1937–38), p. 348. (June 1938.)
[5] Ibid., vol. 9 (1937–38), p. 576. (October 1938.)
[6] Ibid., vol. 9 (1937–38), p. 577. (October 1938.)
[7] Ibid., vol. 10 (1938–39), p. 117. (November 1938.)
[8] Ibid., vol. 10 (1938–39), p. 119. (November 1938.)
[9] Ibid., vol. 10 (1938–39), p. 408. (January 1939.)
[10] Ibid., vol. 11 (1938–39), p. 173. (February 1939.)

Metallurgical research has been largely concerned with the behaviour of materials at high temperatures; in particular, steels for steam power plant are being studied in association with the British Electrical and Allied Industries Research Association. The creep of a series of high-carbon steels is being studied at temperatures of from 300° to 1000° C., and the incidence of graphitization, spheroidization, and the alteration in the lamellar character of the pearlite are receiving particular attention. Other subjects of study include the formation of intercrystalline and transcrystalline fractures in iron and steel, the inter-relation of age-hardening and creep performance, and the influence of internal conditions on the creep behaviour of steels. A research on creep recovery has been continued. In the study of aluminium alloys, age-hardening is receiving much attention, especially in connexion with the aluminium-zinc system, and the X-ray investigation of single crystals of an age-hardening alloy of aluminium and copper has been continued. Other investigations described relate to the cracking of boiler plates, the forging of magnesium alloys, oxides in steel, surface-finish, the structure of graphite, and the residual internal stress in thick-walled hollow cylinders overstrained by internal pressure.

Investigations on behalf of the Radio Research Board include a continuation of the study of the ionosphere, particular attention being given to the effect upon it of solar disturbances. On account of the increased use of ultra-short waves, studies are being made of their propagation along the ground and through the lower atmosphere. Investigations in radio direction-finding have been continued, and the simple rotating-loop direction-finder has been found to be capable of a high instrumental accuracy on wave-lengths between 6 and 10 metres. Other instrumental work includes the development of field-strength measuring apparatus and the study of constant-frequency oscillators.

In a research on lubrication, measurements of static friction have been made by a modified Deeley machine, and it has been possible to obtain consistent values of the coefficient of friction with a large number of substances as lubricants. Of the oils tested so far, fatty oils are best, extreme-pressure oils next, and mineral oils worst. Investigations on journal bearings have been continued and have included some preliminary experiments on the effect of restriction of the lubricant-supply. A study of the extreme-pressure lubricants in the four-ball high-pressure testing apparatus shows marked differences between the behaviour of different types of lubricant. Comparison has been made on the same machine between animal, vegetable and mineral oils, which have also been tested in the oscillating bearing machine and the static friction machine and are now being tested in the journal bearing machine. The results obtained will also provide a useful correlation of the various testing machines. A detailed analysis of the friction between moving metals has been carried out at Cambridge and throws considerable light upon the fundamental

mechanism of sliding ; the area of intimate contact between polished surfaces has also been investigated and is found to be very small. The results of this investigation have important practical implications.

A special study has been made of methods for the detection of toxic gases in industry, and leaflets have been published or are in the press dealing with hydrogen sulphide, hydrogen cyanide vapour, sulphur dioxide, benzene vapour, nitrous fumes, and aniline vapour.

The development and use of X-ray analysis under the auspices of the Committee on the Application of X-ray Methods to Industrial Research has been continued. The structures of electro-deposited metals, cold-worked metals, and various micas have been examined, and both X-ray and electron-diffraction methods have been used in the study of the oxide films formed in the atmospheric corrosion of zinc and iron.

The research on gas-cylinders and containers has included the investigation of various types of welding, the effect of carbon monoxide on steel gas-cylinders, the condition of coal-gas and oxygen cylinders after various lengths of service, and the use of bursting-disk safety devices for gas cylinders.

The Report devotes much attention to the work of research associations, of which the British Cast Iron Research Association, the Iron and Steel Industrial Research Council, the British Non-Ferrous Metals Research Association, the British Refractories Research Association, the British Electrical and Allied Industries Research Association, and the British Colliery Owners Research Association are of particular interest to engineers. The work of these and of a number of other industrial research associations is very fully described.

THE RESEARCH WORK OF THE INSTITUTION OF GAS ENGINEERS.

The organization of the researches carried out by the Institution of Gas Engineers is in some respects similar to that adopted by the Institution of Civil Engineers. Researches are carried out by Technical Committees which are responsible to the Research Executive Committee, and the experimental work is placed in the hands of suitable laboratories of universities, the Department of Scientific and Industrial Research, and other organizations. An Advisory Committee on Research, which represents a very wide range of interests, meets to receive and consider the Annual Report of the Research Executive Committee and to discuss on broad lines the research requirements of the gas industry so as to assist the Research Executive Committee, especially in regard to the trend of technical development ; it has, however, no responsibility for the organization or details of research. The following notes indicate briefly some of

the problems in the production, distribution and utilization of gas that
are at present being studied.

A Joint Research Committee of the Institution of Gas Engineers and
Leeds University is responsible for much of the experimental work that
is carried out. The most important work at present in hand is the con-
tinuation of a long series of investigations on the synthesis of gaseous
hydrocarbons at high pressure. This is of special importance in connexion
with the complete gasification of coal. It has been found that a con-
siderably extended range of gasification is obtained by hydrogenation up
to 900° C., both with coals subjected to preliminary carbonization up to
500° C. and with coals not subjected to preliminary heat-treatment, yields
of from 500 to 600 therms per ton of gaseous hydrocarbons having been
obtained. 70 to 85 per cent. of the carbon of the coals was gasified, the rate
of hydrocarbon-production being well maintained towards the end of the
experiments when only small amounts of residue remained. Similar results
were obtained with high-temperature cokes, i.e. those which had previously
been subjected to a temperature of 800° C. The yield of hydrocarbons
from such cokes below 800° C. was low, but 370 per cent. increase in yield
was attained by further hydrogenation up to 900° C. 530 therms of
gaseous hydrocarbons per ton were obtained with a gasification of 83 per
cent. of the coke. Experiments have also been made on the effect on
gasification of the addition of alkalies, which not only greatly accelerates
the rate of gasification at relatively low temperatures, but also increases
the ability of coke to withstand heat-treatment without losing its reactivity
towards hydrogen and considerably reduces the caking properties of
coals.

The Joint Research Committee is also investigating the Aeration Test
Burner, which has been devised to examine the suitability of gases of
different compositions—though of the same calorific value—for use in
domestic and industrial appliances. The burner is essentially a precision
Bunsen burner calibrated in such a way as to permit the determination of
the precise air-supply necessary for correct combustion of the gas. The
consistency of the readings of the burner over a prolonged period and the
effects of atmospheric pollution and varying humidity on the results have
been studied.

The Gas Works Safety Rules Committee has been particularly con-
cerned with the publication of Voluntary Rules for the Safe Operation of
Purifiers, which were first drafted by the Purifiers Committee. The
reports of the Committee have also dealt with the corrosion of gasholder
sheeting, respirators for carbon monoxide, the testing of gasworks valves,
air raid precautions for gasworks, and other related matters.

The Joint Lighting Committee has given particular consideration to
increasing the efficiency of gas burners, mantles and switches, and a draft
standard specification for Low Pressure Gas Mantles has been completed
by the Committee in collaboration with representatives of the mantle

makers and has been passed to the British Standards Institution as a suggested basis of a new British Standard Specification.

The Liquor Effluents and Ammonia Committee has not recently conducted any special investigations, but researches so far carried out and their application to the various problems of the recovery and utilization of gasworks ammonia and the disposal of effluents therefrom have been summarized in a book entitled " Gas Works Effluents and Ammonia," published by the Institution of Gas Engineers. Frequent inquiries are received by the Committee regarding the disposal of liquor; the general recommendation still holds good that concentration of liquor for further utilization is the best method of disposal. Large-scale investigations of the Committee have proved that gasworks effluents, in admixture with ordinary sewage, can generally be dealt with at a properly-organized sewage-disposal works without difficulty.

The Pipes Committee has recently revised the specification for wrought-iron pipes previously prepared, and meets regularly to consider all problems and information relating to mains and service-pipes that are brought to its notice.

Many problems are being studied by the Refractory Materials Joint Committee, including the action of alkalis, hydrocarbon gases, and slags on refractories. The refractoriness-under-load test, in the form in which the temperature is maintained while the load is increased, is now being studied in its application to silica bricks; although the results show a good deal of variation, average values can be used to discriminate between different brands. An attempt has been made to evaluate the different methods of determining the apparent porosity of refractory materials, and work on jointing cements has been continued.

Communications on research subjects that have been published during the past year include a progress report of experiments at the Fuel Research Station on the complete gasification of coal and methane synthesis; experiments in a modified water-gas plant have shown that sized carbonaceous coals may be successfully gasified, a yield of 205 therms per ton being obtained. Coals having a high content of volatile matter can also be treated, but coals of medium volatile-matter content are too highly caking. In the synthesis of methane for enriching low-grade gas in a two-stage system of complete gasification, experiments with a cobalt-thorium catalyst have given encouraging results. A communication on the hot patching of retorts by blow-pipe spray welding describes a successful method of repairing leaks in continuous vertical retorts while hot, using dry materials and an oxy–coal-gas flame; other papers deal with the production of free-burning coke in continuous vertical retorts, and with results attained with solid fuels, particularly coke, used in open grates for heating living-rooms.

The Institution maintains a Gas Research Fellowship at Leeds University, and a report for 1936–38 deals with the influence of furnace

atmosphere on the scaling of mild steels at temperatures from 1,200°–1,400° C. It is observed that as the liquefying temperature was approached there was a very rapid increase in scale formation, but when once the scale was formed there was only a slow rate of increase with increase of temperature.

In addition to the researches organized by the Research Executive Committee, the Institution of Gas Engineers also gives financial and advisory support to researches undertaken by other organizations. The ventilation of rooms in dwelling-houses, offices, and the like, which is closely connected with the method of heating employed, is being studied with the collaboration of the London School of Hygiene and Tropical Medicine. The combustion-process of town gas is being studied at the Chemistry Department of Cambridge University, and the origin of, and methods of diminishing, noise produced by gas burners are being investigated at the National Physical Laboratory. The stresses in the guide-rails and rollers of spirally-guided gasholders are being investigated in collaboration with the National Physical Laboratory by means of wind-tunnel experiments. Lastly, support is given to an investigation undertaken by the Research Council of the British Iron and Steel Federation, in co-operation with other furnace-using industries, into the fundamentals of furnace design and operation.

The Research Executive Committee issues an annual progress report, which is presented for discussion, together with the individual technical reports, at the Autumn Research Meeting of the Institution of Gas Engineers, which is normally held in London in November of each year.